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Multifactor Productivity in Canada: An Evaluation of Alternative Methods of Estimating Capital Services

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Abstract

This paper examines the effects of alternative specifications of the user costs of capital on the estimated price and volume indices of capital services. It asks how sensitive the results are to the use of exogenous versus endogenous rates of return, to alternate ways of including capital gains, and to whether corrections are made for tax rates. The paper also examines the effect of the various user cost formulae on the measured multifactor productivity growth.

Keywords: multifactor productivity, capital services, rental price of capital

Executive summary

Multifactor productivity growth measures have been developed as summary statistics to measure improvements in the efficiency of the production process—that come from technological progress and organizational change. They do so by comparing actual growth rates in output with the increase in output that would have been expected from an increase in inputs using pre-existing or current production techniques.

At any point in time, existing techniques allow additional factor inputs (labour, capital) that are applied to the production process to produce additional output. Additional factors that are added to the production process multiplied by the existing marginal product of those factors provides an estimate of the expected amount of output in a given period that should have resulted from the use of these factor inputs. If actual output exceeds this, productivity is said to have increased.

Multifactor productivity is measured as the difference between growth in output ΔQ and the increase in output that would have been expected ΔQe because of the use of additional units of inputs (ΔI). This is calculated as the difference in the rate of change of output minus a weighted average of the change in inputs (labour, capital), where the weighted average is calculated to give a proxy for the change that should have occurred if inputs had been as productive as they were in the past.

The weights that are generally used to aggregate changes in a type of factor are the relative shares of each type of factor in the total compensation received by that factor. The unit price of each type of factor is needed to estimate these shares. In the case of prices for labour, the task is relatively straightforward. Transactions are observed continuously in labour markets that can be used for this purpose. In the case of capital, the unit price of capital needs to be estimated. While the price of the capital good is available, the price of the services that the capital good should command, when it is used over a period that is shorter than its length of life, is not usually observed and needs to be inferred.

Economic theory is used to suggest a formula that can be used to infer the price of capital. The user cost of capital can be thought of as the price that a well functioning market would produce for an asset that is being rented by an owner to a user of that asset. That price would comprise a term reflecting the opportunity cost of capital, a term reflecting the depreciation of the asset, and a term reflecting capital gains or losses from holding the asset. This formulation requires data on the rate of return, depreciation, capital gains from holding assets, tax rates on capital, and the price of the asset.

In this paper, we discuss the methodology that has been employed to estimate rental prices of different assets and the problems involved in their implementation. We then present a set of alternate estimates of multifactor productivity growth using different methods to generate the required data on the rate of return, the depreciation rate, capital gains and tax rates.

Several issues that are raised in the literature are discussed and addressed in turn in this paper. The first is whether rates of return should be calculated exogenously or endogenously. The second is the source of depreciation rates. The third is whether and how capital gains should be included in the user cost of capital formula. The fourth is how tax rates are included in the formula.

The differences in the estimates of multifactor productivity (MFP) that have been provided in this paper outline the extent to which alternate assumptions matter. The alternate methods considered here yield a range of estimates. Inclusion of capital gains has a significant impact not only on the estimated growth in capital services but also on its volatility. Our method of including capital gains in most of the variants investigated in this paper is the conventional one—that involves calculating the rate of changes in the prices of investment goods over time.

There is considerable volatility in the capital gains term when this is done. Since this term implicitly captures the expectation of capital gains and the volatility in this term is large, it is difficult to choose a method of averaging that properly incorporates expectations.

Since it is essentially arbitrary whether we include or exclude capital gains, we can place a bound on our estimates of the growth in multifactor productivity by comparing the difference between estimates with and without this term. For the period 1981 to 2001, these differences are 0.1 percentage point or about 23% to 30% of the MFP productivity estimate. For the period 1961 to 1981, the ranges are smaller and are around 4% to 10% of the MFP estimate.

We also find that ignoring corporate taxes leads to an upward bias in the growth of capital services for the endogenous rate method for both the 1961 to 1981 and 1981 to 2001 periods.

The other major choice that has to be made is between endogenous as opposed to exogenous rates. The Canadian data do not show large differences in the actual rate of returns—where the endogenous rate is calculated from National Accounts industry data and the exogenous rate of return is calculated from a weighted average of the equity and the bond markets. It is worth pointing out that the differences between the two MFP estimates using each are not large. The endogenous MFP rate is about 12% to 14% below the exogenous MFP rate—within the margin of error that one would expect from misstating the exogenous rate of return by 25%.

The quality of any statistical program depends upon the extent to which the summary statistics are fit for use. Meeting this objective requires that agencies give users guidance on appropriate use. Generally, only point estimates of MFP are provided by statistical programs. The effect of using alternate assumptions for the production of summary statistics is rarely investigated. This paper addresses this gap for MFP estimates.

Doing so is important when users employ MFP measures in cross-country comparisons. Users need to be aware that the MFP estimates are point estimates with a confidence interval around them. That confidence interval cannot easily be calculated using classical statistical techniques. But we can give users an idea of how our estimates would change if we employed alternate assumptions.

We have shown here that the point estimates that we produce could very well vary by 20%, either because of the choice of the endogenous over the exogenous technique or because of the way in which capital gains are measured. When users employ MFP estimates for cross-country studies, these types of errors should be kept in mind.

1. Introduction

Measuring economic performance involves comparisons in terms of output, inputs or productivity measures across industries and over different time periods. Productivity measures track changes in outputs and inputs so as to infer the magnitude of efficiency gains in the production process.

Labour productivity measures provided the core output of most productivity programs of statistical agencies until the 1980s. More recently, multifactor productivity programs have been introduced that consider not just labour but also capital and that allow richer pictures of the production process to be considered when estimating efficiency gains.

The more comprehensive estimates produced by multifactor productivity (MFP) programs come at a cost. The concept is more difficult to explain to users and is based on more complex underpinnings. Alternate formulations exist and produce different answers.

One area in which practice has not coalesced around one method is that of capital measurement. Here there is debate about how to measure aggregate capital services and how to capture growth in capital services over time. An important part of this debate revolves around how the price of capital should be estimated.

Prices for other factors are readily observed in markets. While the price and volume of labour and intermediate inputs are observed as a result of market transactions, the rental price and volume of capital services are not directly observed. Wage rates come from labour force surveys. Output and input prices come from price surveys. The market for capital services (the rental market for capital services) is however less developed and statisticians generally derive the price of capital services analytically.

Jorgenson and Griliches (1967) developed a method for measuring the price and volume of capital services that is based on the economic theory of production. The concept of capital services and capital rental prices is now well established with further developments due to Hulten (1990), Diewert (2004), and Schreyer, Diewert and Harrison (2005). The issues in measuring the price and volume of capital services are mostly empirical, the most important of which involves: (1) the choice of expected rate of return, (2) the choice of expected capital gains, and (3) the treatment of corporate tax provisions in the user cost estimation.

In this paper, we briefly discuss several alternatives that are available and then present evidence using Canadian data on the practical effect of using alternate approaches.[1] We are interested in whether there are strong theoretical grounds for one formulation over another, whether there are practical grounds for making choices and finally whether it matters much for the final estimates.

1. A number of other studies have examined the effects of alternative capital rental price formulae on the capital services and MFP estimates. See Harper, Berndt and Wood (1989), Schreyer (2004), Diewert (2004), and Harchaoui and Tarkhani (2003).

The outline of the paper is as follows. In next section, we present a brief description of multifactor productivity estimates. In section 3, we discuss some of the issues that revolve around estimation of the price of capital. In section 4, we discuss the data used for the analysis. The results of alternate estimation techniques are presented in section 5. A conclusion follows in section 6.

2. Measuring multifactor productivity

Productivity gains occur because producers manage to find more efficient ways of producing goods. These gains originate from many sources—from technical change, organizational change, and from exploiting scale economies. We are better off today because labour today can produce much more per person that it could 100 years ago.

Multifactor productivity growth measures have been developed as summary statistics to measure the amount of this progress. They do so by comparing actual growth rates in output with the increase in output that would have been expected from an increase in inputs using pre-existing or current production techniques.

At any point in time, existing techniques allow additional factor inputs (labour, capital) that are applied to the production process to produce additional output. Additional factors that are added to the production process multiplied by the existing marginal product of those factors provide an estimate of the expected amount of output in a given period. If increases in actual output exceed this, productivity is said to have increased.

Multifactor productivity is measured as the difference between growth in output ΔQ and the increase in output that would have been expected ΔQe because of the use of additional units of inputs (ΔI).

1) $$MFP = \Delta Q - \Delta Qe = \Delta Q - F(\Delta I),$$

where F represents the increase in output using existing techniques that we would expect from the increase in the inputs that occurred over a given period.

The microeconomic theory of the firm uses a ‘production function’ to formally describe the relationship between the services of inputs and output.[2] In order to base multifactor productivity measures on production theory, economists have specified a twice differentiable production function relating output and factors of production (labour and capital).

(2) $$Q = F(X_1, X_2, ...X_n, t),$$

where X_i represent the *i*’th input and *t* is time.

2. Alternately, theorists sometimes start with a cost function to derive a measure of multifactor productivity.

The total differential of (2) with respect to time is

$$(3) \qquad \frac{dQ}{dt} = \sum \frac{\partial F}{\partial X_i} \frac{dX_i}{dt} + \frac{\partial F}{\partial t}.$$

The contribution that increases in labour or capital would be expected to make to output growth is just the marginal product of labour (capital) multiplied by the change in labour (capital) devoted to production. Dividing both sides of (3) by Q and defining

$$(4) \qquad M\dot{F}P = \frac{1}{Q} \cdot \frac{\partial F}{\partial t}$$

and recognizing that $\frac{dX_i}{Qdt} = \frac{X_i}{Q} \dot{X}_i$ gives

$$(5) \qquad M\dot{F}P = \frac{dQ}{dt} \cdot \frac{1}{Q} - \sum \frac{\partial F}{\partial X_i} \frac{dX_i}{dt} \frac{1}{Q} = \dot{Q} - \sum \frac{\partial F}{\partial X_i} \frac{X_i}{Q} \dot{X}_i .$$

Now the first order conditions for profit maximization are

$$(6) \qquad \frac{\partial C}{\partial X_i} = \frac{\partial F}{\partial X_i} .P ,$$

where *P* is the price of *Q* and *C* is total cost ($\equiv \sum P_i X_i$).

Recognizing $P_i = \frac{\partial C}{\partial X_i}$ and substituting into (5) gives

$$(7) \qquad M\dot{F}P = \dot{Q} - \sum \frac{P_i X_i}{PQ} \dot{X}_i = \dot{Q} - \sum s_i \dot{X}_i ,$$

where s_i is the factor *i*'s share in output (PQ).

If the production function is characterized by constant returns to scale and prices of factors (labour and capital) equal their marginal revenue product, then the share of labour in gross domestic product (GDP) and the share of capital in total product just exhaust total GDP. If not, then the formula has to be modified to

$$(8) \qquad M\dot{F}P = \dot{Q} - \sum \varepsilon_{cy}^{-1} \frac{P_i X_i}{C} \dot{X}_i ,$$

where ε_{cy} is the measure of the scale of production (the cost elasticity of output).

This approach therefore allows the statistician to approximate the marginal productivity term in (5) using factor prices and the share of a factor in output. But as Berndt (1990) has stressed, we are using these as proxies for "the realized marginal products." For it is this concept that allows us to measure what output would have been expected, based on existing production techniques.

Multifactor productivity growth then can arise from outward shifts in the aggregate production function brought about by technological change, and—under certain conditions—the latter can be measured by changes in the multifactor productivity measure alone. When these conditions do not hold, alternative methods are used to separate the technological shift component from other factors, like the exploitation of scale economies that are at work.[3]

Estimates of multifactor productivity from (7) require measures of the change in output (*Q*), capital (*K*), and labour (*L*) and factor shares. In a world where all factors of a certain type (labour or capital) have the same marginal product, changes in the factor may be estimated by simply summing the value of all components and calculating changes of the total over time. But factors (either workers or types of capital assets) may differ in terms of their marginal product and then it is inappropriate to simply sum the factors without weighting each separately. If there are *m* types of factor *i*, each with a different marginal product, then the appropriate formulae for estimating the effect of a change in a factor is

(9) $$s_i \dot{X}_i \equiv \sum_{k=1}^{m} s_{ik} \dot{X}_{ik} ,$$

where s_{ik} can be approximated by the share of total output that goes to each type of the factor *i* and

(10) $$s_i = \sum_{k=1}^{m} s_{ik} .$$

The rate of growth of $\dot{X}_i$ that is needed in equation (7) is

(11) $$\dot{X}_i = \sum_{k=1}^{m} (s_{ik} / s_i) \dot{X}_{ik} .$$

As long as s_{ik} / s_i are different across individual factors, the rate of growth of $\dot{X}_i$ produced by equation (11) will differ from the rate of growth that is derived from simply summing all factors (number of workers, total value of assets) and calculating the rate of growth from the total. This difference stems from differences in the rate of growth of different assets ($\dot{X}_i$) that concomitantly change the composition of the asset bundle. As a result, this difference is often referred to as the 'composition' effect. Since changes in the composition of the bundle affect the relative amount of low marginal product as opposed to high marginal product factors, it is also sometimes referred to as the 'quality' effect.

3. See Baldwin, Gaudreault and Harchaoui (2001) for an illustration of the parametric approach to productivity measurement.

The appropriate weights then to aggregate changes in a type of factor are the relative shares of each type of factor in the total compensation received by that factor. In order to estimate these shares, we need to calculate the unit price of each type of factor. In the case of prices for labour, the task is relatively straightforward. Transactions are observed continuously in labour markets that can be used for this purpose. In the case of capital, we need comparable prices. While the price of the capital good is available, the price of the services that the capital good yields, when it is used over a period that is shorter than its length of life, is not usually observed and needs to be inferred.

In this paper, we discuss the methodology that has been employed to estimate rental prices of different assets and the problems involved in their implementation.[4] We then present a set of alternate estimates of multifactor productivity growth using alternate assumptions.

The issues that are approached here are of fundamental importance in the measurement of capital input used by the multifactor productivity framework. The manner in which the components of the user cost of capital are measured makes them particularly vulnerable to error-in-variable problems. Most practitioners provide point estimates of multifactor productivity with little guidance for users on the nature of the confidence intervals that need to be used.[5] One of the criteria used by Statistics Canada to assess the quality of its statistical program is the extent to which adequate guidelines are provided so that users can make informed judgements about the adequacy of a particular statistic for a particular use. By examining the impact of making alternate assumptions and assessing their impact on the final estimates, this paper is aimed at informing debate in this area.

We first review the practical issues that need to be resolved in employing the rental price formulae used in the empirical literature. This results in a set of rental price measures that we then evaluate in the context of the multifactor productivity growth accounting framework.

3. Measuring the rental price of capital

Capital is, of course, just one type of input. However, capital goods do not neatly conform to the simple production model that involves other assets. Among other things, capital assets are not consumed immediately in production as are material inputs. Capital goods are exhausted over several time periods and any price that is used must recognize that the asset provides services over several time periods. A measure of capital input which would be consistent with production theory is therefore the quantity of the *flow of services* provided by capital goods over the time period being used for analysis.

To address this measurement problem, Jorgenson (1963), Hall and Jorgenson (1967), Griliches and Jorgenson (1966), and Jorgenson and Griliches (1967) developed the notion of an asset-

4. The same issue is relevant when it comes to estimating the change in hours-worked.
5. See Harper, Berndt and Wood (1989), and Schreyer, Diewert and Harrison (2006) for other exercises that examine some of these issues.

specific "user cost of capital", sometimes referred to as the rental price of capital. This price then can be used to aggregate heterogeneous capital assets.

The user cost of capital can be thought of as the price that a well functioning market would produce for an asset that is being rented by an owner to a user of that asset. That price would comprise a term reflecting the opportunity cost of capital (r_t) (either the opportunity cost of using capital or the financing costs), a term reflecting the depreciation of the asset (δ), and a term reflecting capital gains or losses from holding the asset (reflecting changes in the market price of an asset, $q_t - q_{t-1}$). Jorgenson and Griliches (1967) show that the formula for the rental price of a unit of capital that costs q is

$$(12) \qquad c_t = q_{t-1} r_t + q_t \delta - (q_t - q_{t-1}).$$

This user cost approach has important advantages over the capital stock approach, which uses acquisition prices to weight different assets. Weighting assets by their user cost, which approximates the marginal revenue product in a competitive equilibrium, effectively incorporates differences in the productive contribution of heterogeneous investments as the composition of investment and capital changes. In this case, changes in aggregate weighted capital input have two distinct components—changes in the quantity of capital of a given type, and changes in the composition of the various types of assets with different marginal products and user costs. This second effect —arising from the change in the importance of capital types in the aggregate capital stock—is referred to here as the composition effect resulting from changes in the bundle of capital assets. The increase in the weighted bundle will be higher than in the simple aggregate when assets with higher rental prices (representing higher marginal products) are increasing at faster rates. Weighting explicitly captures substitution between heterogeneous assets in response to changing relative prices, or biased technical change.

To take into account taxes, Christensen and Jorgenson (1969) developed the following user cost formula c_{kt} for the kth capital asset type in period t

$$(13) \qquad c_{kt} = \left(\frac{1 - u_t z_{kt} - ITC_{kt}}{1 - u_t} \right) [q_{kt-1} r_t + q_{kt} \delta_k - q_{kt-1} \pi_{kt}] + \phi_t,$$

where ϕ_t is the effective rate of property taxes (nominal valued taxes assessed on the nominal stocks of land and structures), u_t is the corporate income tax rate, z_{kt} is the present value of depreciation deductions for tax purposes on a dollar's investment in capital type k over the lifetime of the investment, ITC_{kt} is the rate of the investment tax credit, and $\pi_{kt} = \frac{(q^*_{kt} - q_{kt-1})}{q_{kt-1}}$ is the expected capital gains.

The user costs in (12) and (13) are formulated in terms of nominal rates of return and nominal rates of asset price inflation. Schreyer, Diewert and Harrison (2005) suggest that it is more practical to work with real rates as real rates are much more stable than nominal rates, particularly in high inflation periods. The user cost formula (13) can be modified to obtain a formula that is based on real rates:

$$(14) \qquad c_{kt} = \left(\frac{1 - u_t z_{kt} - ITC_{kt}}{1 - u_t}\right)\left[q_{kt-1} r_t^* + q_{kt}\delta_k - q_{kt-1}\pi_{kt}^*\right] + \phi_t ,$$

where r_t^* is the real rate of return and is calculated as the nominal rate of return deflated by an overall price index (such as the gross domestic product [GDP] deflator or the consumer price index [CPI]), and π_{kt}^* is real capital gains of the *k*th asset.

Denoting ρ_t the overall inflation rate, we can write the real rate of return and the real rate of capital gains as follows:

$$(15) \qquad r_t^* = r_t - \rho_t, \text{ and } \pi_{kt}^* = \pi_{kt} - \rho_t .$$

In this paper, we will use the user cost specification (14) that is based on real rates. To obtain a measure of the overall inflation rate ρ_t, we construct a 5-year centered moving average of the rate of change of the consumer price index.

These rental prices of capital are then used to create the weights s_k for aggregating the growth in individual asset-specific capital stocks (K_k) using equation (11):

$$(16) \qquad \dot{K} = \sum_{k=1}^{m} s_k \dot{K}_i \text{ , and } s_k = c_k K_k / \sum_k c_k K_k ,$$

where the weights s_k are the share of each asset in total capital costs.

While Jorgenson’s pioneering efforts have provided the framework needed to overcome the lack of directly observable and measurable prices of capital services, providing a link between the model’s theoretical structure and its application has been more difficult. In particular, there is a considerable difference between the rental price as a theoretical paradigm and its real world empirical application. The next section of this paper discusses some of the issues that have to be resolved in order to bridge this gap.

3.1 Issues in estimating the rental price of capital

Analysts who calculate rental prices of capital services face several choices—with regards to the expected rate of return, depreciation rates, expected capital gains, expectations, and finally whether to include tax parameters in the formulae. We start with a brief discussion of the choices facing the analyst in each of these areas.

3.1.1 *Exogenous versus endogenous rates of return*

Two main alternatives are available for estimating the rate of return on capital: rates that are calculated endogenously from the System of National Accounts and rates that are chosen exogenously from observed market rates.

Rates that are calculated endogenously make use of data from the national accounts on estimates of capital stock and capital income that consists of gross operating surplus and the portion of mixed income attributable to capital. Using the formulae for the cost of capital, and recognizing that capital income just pays for capital services allow us to solve for the rate of return in (17)

(17) $$R_t = \sum_k c_{kt} K_{kt} \text{ ,}$$

where R_t is capital income, K_{kt} is the capital stock of the *k*th asset and c_{kt} is the user cost of the *k*th asset.

Substituting the user cost of capital c_{kt} from equation (14) in equation (17), we obtain the after-tax rate of return:

(18)

$$r_t^* = \frac{R_t + \sum_k q_{kt-1} T_{kt} K_{kt} \pi_{kt}^* - \sum_k q_{kt} T_{kt} K_{kt} \delta_k - \sum_k q_{kt-1} K_{kt} \phi_t}{\sum_k q_{kt-1} T_{kt} K_{kt}}, \text{ and}$$

$$T_{kt} = \frac{1 - u_t z_{kt} - ITC_{kt}}{1 - u_t}.$$

The capital income R_t as measured in the System of National Accounts excludes capital gains. In equation (18) for calculating the real rate of return, the aggregate real capital gains term $\sum_k q_{kt-1} T_{kt} K_{kt} \pi_{kt}^*$ enters with a positive sign so as to augment capital income reported in the System of National Accounts. The estimated real rate of return in (18) therefore includes the expected real capital gains.

Alternatively, the rate of return can be taken from other sources—a rate of return observed in financial markets, for example. Here, there are several choices—a risk-free rate of return such as a government bond rate, a corporate debt rate that takes into account the risk of the business sector, or a weighted average of corporate debt and corporate equity rates that recognizes that the corporate sector is financed by a mixture of debt and equity.

Canada, Australia and the United States use the endogenous rate of return in their productivity accounts. In the past, the Organisation for Economic Co-operation and Development (OECD) used the approach of taking an average of factor shares derived from the experience of member countries, which falls in the endogenous camp, at least in spirit. More recently, the OECD has adopted an exogenous rate of return. (Schreyer, 2004) And a recent background paper prepared for the Canberra group (Schreyer, Diewert and Harrison, 2005) proposes that exogenous rates be given serious consideration for estimating capital services.

The advantage of using the method that employs endogenous rates is that it is provides a fully integrated set of accounts. The surplus is taken directly from the National Accounts that provides the underlying data for the productivity accounts. Capital is directly estimated from the investment flows that are also part of the System of National Accounts. In Canada, investment flows are integrated with the input-output tables and are thus, consistent with output at the industry level. Equally important, the assumptions that are required to make use of the surplus in estimating capital services are fully compatible with the assumptions that underlie the non-parametric productivity estimates—that of a fully competitive economy with a production process subject to constant returns to scale. While these assumptions may not hold exactly, the issue that statisticians must face is whether relaxing them markedly affects the productivity estimates.[6]

Choosing an exogenous rate of return allows the assumption of constant returns to scale to be relaxed. And it does not require that the assets used completely exhaust capital income, thereby recognizing that some assets may be excluded in existing estimates. It also allows an analyst to presume that the economic system is not perfectly competitive and that the corporate surplus may include more than just the cost of capital services—for example, monopoly profits.

Since the use of an exogenous rate of return does not guarantee that the corporate surplus is completely exhausted, it permits the estimation of a residual (the difference between corporate surplus and capital services). This difference could arise because of monopoly profits. It could arise because the list of factors that is included in the multifactor productivity estimates is incomplete (for example, assets like land, inventories, natural resources or intangibles are often excluded). It could arise because there are economies of scale and therefore, paying factors their marginal revenue product does not completely exhaust total product.

It should be noted that calculating a multifactor productivity growth rate with any one of these influences removed should be expected to produce different estimates of multifactor productivity. A standard non-parametric estimate of multifactor productivity, that assumes no scale economies, includes any productivity-enhancing improvements of increasing scale in the estimate of productivity gains. An estimate that removes it, is attempting to “purify’ the estimate of the shift in the production function of this scale effect and should be lower if the scale effect is important. The degree to which the latter is useful depends not just on the statistical accuracy with which the exercise can be performed but also the meaningfulness of the exercise. Multifactor productivity estimates are regarded as measures of increased technical efficiency. If improvements in technology are needed to exploit scale economies, it is not very meaningful to try and separate scale effects from technology effects.

6. In Baldwin, Gaudreault and Harchaoui (2001), we show that relaxing the assumption of constant returns to scale and using parametric rather than non-parametric estimates of productivity do not dramatically affect our estimates of multifactor productivity.

It is also noteworthy that the multifactor productivity estimate that comes from using an endogenous rate and too few assets is not necessarily inaccurate. In fact, it will be approximately correct if the rate of growth of the omitted asset is the same as the average growth rate of included assets. A multifactor productivity estimate that uses an endogenous rate and attributes all capital surplus to a subset of assets may be closer to the correct estimate than one that uses an exogenous rate and misses some assets. The latter will clearly be biased upwards. The former will be approximately correct if the omitted assets make up a small share of the total capital stock and their rate of growth is about the same as the bundle of assets that are captured.

While using the exogenous rate overcomes several potential problems, it gives rise to others. The problem with using the exogenous rate is that it is not obvious what rate should be used. And choice of an incorrect rate will lead to an error in the estimates of multifactor productivity.

In the case of an exogenous rate, there is a wide range of rates that have been suggested—from short to long rates, from lending to borrowing rates (Diewert, 1980). The interest rate in the cost of capital formula should reflect risk-adjusted rates of return (since it is these that govern investment decisions). This requires a variation in the return by industry or by asset to reflect varying degrees of riskiness.[7] This problem, in turn, requires that the analyst make use of information that would help to adjudicate differences in risk. Data exist in the input-output tables in the National Accounts that allow us to estimate long-run differences in profitability by industry that could be used along with an average exogenous rate of return to allow for industry-specific differences in risk. But when this is done, there may, in the end, be little difference between the rates yielded by an endogenous and an exogenous system.

In this paper, as in the official Canadian Productivity Accounts, we use capital income from the Canadian National Accounts to derive the internal rate of return. Capital income is defined here as current dollar gross domestic product except for labor compensation (wages, salaries, supplementary compensation, and a portion of proprietors income attributable to labour). Capital is taken from the productivity accounts database of Statistics Canada. It is created from investment flows using the perpetual inventory method.

For the exogenous rate of return, we have used a weighted average of debt costs and the equity rate of return, where the weights are the proportion of debt and equity that is used to finance business capital.[8] For the debt rate, we have used average 10 year Government of Canada bond yield. For the equity rate, we have used the rate of return on equity as derived from the gain in the index of the Toronto Stock Exchange plus the dividend yield.[9] The resulting exogenous rates of return are inclusive of the overall inflation rate and thus represent the nominal rates of return. These nominal rates are then deflated by the consumer price index. The resulting series of real exogenous rates are averaged over the period 1961 to 2001 to yield a constant rate of return of 5.1%. For the user cost specification (14) based on the exogenous rate of return, we will set the real rate of return r_t^* to a constant, 5.1%.

7. See Schreyer, Diewert and Harrison (2005, p. 43) who stress that practitioners should therefore use industry-specific rates of return that reflect that some investment in fixed capital is riskier than others.
8. These proportions are taken from the Industrial Organization and Finance Division of Statistics Canada.
9. See Canadian Economic Observer, series 122620 and 122628.

In comparing this rate to that derived endogenously, which is the rate of return gross of expected capital gains, we have to decide whether the exogenous return reflects the rate that is earned net or gross of real capital gains. If it is net, then we must add in real capital gains to attain the rate of return gross of real capital gains. But if the rate already includes real capital gains, there is no need to do so. In our case, we have used the yield on equity that is produced by the market for capital. It can be argued that the rate that this market yields already incorporates the real capital gains that investors expect to accrue to firms and therefore, capital gains do not have to subtracted. But in this paper, we will continue to subtract the capital gains as is so frequently done, all the while providing results using the methodology that treats our exogenous rate as net of capital gains and losses.

3.1.2 Depreciation rates

Depreciation rates in the rental cost of capital are meant to reflect the cost that a lessor of an asset would have to recover from the use of an asset—costs that arise from wear and tear of the asset and other losses in value such as obsolescence.

These rates are derived from the age profile of used asset prices.[10] Obtaining the depreciation profile from the age-price profile is in keeping with the spirit of the rental cost of capital—it is the price that would have to be charged by a lessor to cover the costs associated with the fact that the value of the asset would decline after use. Some try to draw a distinction between the different causes of loss—from wear and tear, and from obsolescence. But our technique and the data available do not permit these distinctions to be drawn.[11]

The depreciation rates that we derive are *ex post*. That is, they reflect realizations in markets and not expectations. However, Statistics Canada also collects data from its investment survey on expected length of life from managers when they first make investments. A comparison of the differences in the depreciation rates derived from *ex ante* estimates of length of life and the *ex post* rates indicates that there is little difference between the two.[12]

3.1.3 Capital gains

The capital gains component of the rental price formula has and still leads to controversy. There is little disagreement that, on theoretical grounds, capital gains should be included in the rental price of capital. A lessor of capital will charge a lower price if a capital gain is expected by the end of the holding period or a higher price if a capital loss is expected.

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10. See Statistics Canada, Depreciation Rates for the Productivity Accounts (2007).
11. The large database on used asset prices, developed from survey data, has only age of asset, original price, and the price realized at disposition. It does not contain the vintage of the capital stock, only its age and therefore we cannot separately estimate obsolescence.
12. Schreyer, Diewert and Harrison (2005) argue for *ex ante* rates of depreciation when exogenous rates of return are used in the rental price and *ex post* depreciation rates when the endogenous rate of return is used. Since the two are so similar, we make use of only one estimate of depreciation here.

Nevertheless, there is some disquiet among practitioners when it comes to including a term for capital gains and losses. Harper, Berndt and Wood (1989, p. 340) discuss this issue noting Denison's early criticism of the original Jorgenson and Griliches (1967) formulation (Denison, 1969). But their discussion essentially revolves around the time period that should be used to derive an expectation of the gain that is to be expected, noting capital gains are highly erratic from year to year. They then suggest that rather than using the actual gains, a moving average might be more appropriate.

Giving rise to unease among practitioners is the size of the actual capital gains and the source thereof. Harper, Berndt and Wood (1989, p. 347) report very large capital losses per year for many years for specific industries. This partially arises because the price of machines and equipment has gone up more slowly than the price of all goods, on average,—because the latter are dominated by services with higher than average rates of growth in prices. And volatility in capital gains is not likely to be matched in the short run by changes in the marginal product of capital because of long gestation periods for capital projects.

It is also not clear whether there are ways that holding-period gains arising from differential rates of inflation can be harvested—especially for investment goods. This concern revolves around the level of transaction costs that must be incurred in selling investment goods. Because many assets are firm specific, they lose a considerable portion of their value when transferred—especially if they are transferred to alternate uses. The value of steel in a petroleum refinery is worth considerably less as scrap than it cost to assemble it into refining towers. If there is no inexpensive way to realize capital gains, changes in asset prices derived from price indices are not a very useful way to measure the capital gains component of the rental price of capital. It is therefore not clear that the actual asset price series provide accurate estimates of the rate of return that should be expected from capital gains.

In this exploration, we therefore make use of several alternatives to test the sensitivity of our estimates of multifactor productivity (MFP) to the treatment of capital gains. In the first case, we include instantaneous capital gains—what others have referred to as the myopic case. We then calculate a 5-year centred moving average. Finally, we exclude them altogether, which is appropriate if the transaction costs of liquidating assets essentially offset potential gains due to price changes.

3.1.4 Expected values

The estimate of multifactor productivity is just the increase in output above the increase that would have been expected from the increase in inputs, using existing production techniques and existing scale of plant. As outlined previously, theory is used to help us find proxies for the marginal revenue product that should be used for this purpose. When a factor is paid its marginal revenue product, we can use the value of the cost of the service of inputs—wage rates for labour and the rental price of an asset.

To do so, we need to recognize that contracts have to be entered into before production decisions occur—or that firms will need to predict what their factor costs are before they can act to adjust factor inputs to bring about the equality between the factor's cost and its marginal product. And that over some periods, this equality will not hold. Neither labour nor capital markets can be adjusted instantaneously. Average wage rates in any period will depend on the portion of hours that are put in at “standard” time and the amount of overtime that is paid a premium. Firms must predict how much of each will occur before a period occurs in order to make decisions on how many workers will be hired—unless hours in each category can be adjusted instantaneously.

The decision is even more complex when it comes to capital because the adjustment period is often longer because of the lumpiness and the length of life of capital. Since investments are lumpy, firms have to introduce more than just incremental capacity and thus face a longer time horizon when equating factor costs and marginal product. This, in turn, complicates the financing decision in a world of changing interest rates.

The analyst then has to decide on a time horizon over which an expectation is likely to be formed. Even then, it is critical to remember that in the short run, it may not be possible for a firm to bring into equality its expected cost of capital with the marginal revenue product. For example, a sudden increase in the expected cost of capital may not be followed immediately by any change in the marginal revenue product of capital if most investments that are brought on-line in subsequent periods resulted from previous decisions.

Differences in the way in which these considerations are handled by researchers reflect differences in assumptions about how the world operates. Most advocate some averaging process. Those who choose a very long-run period, implicitly or explicitly, believe that either expectations change very slowly, or that responses of marginal revenue products are slow. Of course, there may be little to choose between lag operators of different lengths if the real rate of return is more or less constant.

We handle the issue of expectations and changes in rates of return here in two different ways. For the exogenous rate, we take an average of the real rate over the entire period. In effect, we are assuming that expectations and real marginal revenue products are constant over time. For the endogenous rate, we use the annual realized value and implicitly assume that annual changes are reflected in expectations and brought about by changing investment decisions. This gives us two series for rates of return—one derived from the national accounts system and one derived exogenously. By comparing the two, we ask whether two quite different assumptions about the way in which the world operates are important when it comes to deriving estimates of multifactor productivity growth.

3.1.5 Tax parameters

The rental cost of capital formula requires us to estimate a number of tax parameters. The method that is used in Canada is discussed in Harchaoui and Tarkhani (2003). This formula is costly to implement and therefore sometimes ignored.[13] Whether the omission is important will depend upon whether the tax code treats different assets and industries differently, whether the difference between the after-tax and the pre-tax rates differs across industries.

13. Schreyer, Bignon and Dupont (2003).

To examine the effect of including the tax parameters, we therefore estimate capital services both with and without the tax parameters.

3.2 Alternate specifications of capital rental price formulae

To examine the effect of alternative approaches to the estimation of capital services and multifactor productivity, we compare essentially the first six alternatives that are summarized in Table 1. The first three (M1 to M3) make use of a rate of return estimate that is calculated endogenously from the Productivity Accounts of Statistics Canada.

Table 1 Alternative specifications of capital rental price formulae

	Real capital gains		
	Yes, not smoothed	Yes, smoothed	No
User cost with tax parameters			
Endogenous rate of return	M1	M2	M3
Exogenous rate of return	M4	M5	M6
User cost without tax parameters			
Endogenous rate of return	M7	M8	M9
Exogenous rate of return	M10	M11	M12

Note: A variant M5* treats the estimated exogenous rate as net of capital gains.
Source: Statistics Canada, Canadian Productivity Accounts.

The user cost specifications M1 and M2 include real capital gains measured from changes in the asset prices. M3 does not use changes in used asset prices.

M1 is the specification that measures capital gains as the instantaneous rate of price change with no smoothing. It is the appropriate alternative if the expectation of businesses and their investment patterns adjust instantaneously to all the components of the user cost of capital, since it consists of the rate of return appropriate to a particular year and the capital gains term for that year. M2 is the alternative with the capital gains term smoothed—allowing for instantaneous adjustment only to the internal rate of return but with a lag to the capital gains term. M3 is the user cost of capital with the capital gains term as measured by the change in asset prices removed. While all three methods use the period rate of return as derived from the Accounts, there is very little period-by-period variation in the real rate—thereby removing potential problems with this rate if adjustment to volatile capital prices is difficult in an economy.

The second three alternatives (M4 to M6) parallel the first three, but use the exogenous rate of return. In each case, the rental price of capital is calculated by subtracting capital gains from the exogenous rate of return. This presumes that the exogenous rate of return includes capital gains. This may not be the case where equity rates are used to derive the exogenous rate, as they are in this paper. We therefore also discuss an alternative where we treat this rate as having taken into account capital gains—as being calculated net of capital gains—in the same way that the endogenous rate is calculated. When we do so, we add the aggregate real rate of capital gains to the real constant exogenous rate to obtain the gross internal rate of return. We refer to this variant as M5*.

The first six alternatives in Table 1 account for the effects of corporate income tax, capital consumption allowance, investment tax credits and property taxes on the user cost of capital assets. The last six alternatives in the table ignore the effects of tax parameters. At the second stage, we compare the six alternatives with and without consideration of tax parameters.

4. Data

The data for this exercise are taken from the Canadian Productivity Accounts (CPA) maintained by Statistics Canada. The CPA are an integral part of the Canadian System of National Accounts (CSNA), particularly, the Input-Output tables (IOT) and the Income and Expenditures Accounts (IEA).[14] Productivity measurement requires information on prices and quantities of the flow of commodities produced and purchased by various industries, the purchase of durable goods by categories of final demand and the compensation of primary inputs, all of which are available on a consistent basis from the CSNA. The Productivity Accounts make use of the data on gross domestic product (GDP) from the expenditure side and the input-output tables that are consistent with one another and then create a matched database for labour and capital inputs that is integrated with the remainder of the data to create a master industry database—KLEMS (capital, labour, energy, material and services)—that can be used for analytical purposes.

The aggregate output data for this exercise are built up from industry estimates for the business sector.[15] Measurement of productivity at the industry level by the CPA uses prices and quantities available from a complete set of Input-Output tables. This set of accounts is benchmarked to the rest of the accounts, including aggregate GDP as measured by the expenditure side of CSNA. The IOT provide data in constant chained dollars and current prices for gross output, intermediate inputs and value added.

The CPA also contains labour estimates that are constructed from various sources within Statistics Canada that accord with the recommendations of SNA 1993 and that are consistent with the data that are produced by the production accounts. Estimates of jobs and hours-worked are produced at a detailed industry level and by class of workers (see Maynard, 2005). Hours-worked is the base measure used for productivity estimates because it represents a better measure of labour input than employment. The hours-worked measure captures changes in overtime worked, standard weekly hours, leave taken, and changes in the proportion of part-time employees.

Growth in labour input is calculated as weighted averages of growth in different classes of workers in order to take into account differences in the marginal revenue products of different classes of workers. Details on the construction of the labour data can be found in Gu et al. (2003). The demographic groups include 112 different types of workers, cross-classified by class of workers (employee, self-employed or unpaid), age (15 to 17, 18 to 24, 25 to 34, 35 to 44, 45 to

14. For more information on the Canadian Productivity Accounts, see Baldwin and Harchaoui (2005).

15. Recent work that implemented this approach include Jorgenson (2001) for the U.S. economy; and Jorgenson and Yip (2001) for international comparisons, and Harchaoui et al. (2004) for a Canada-U.S. comparison of economic growth and productivity performance.

54, 55 to 64, 65+), and education (0 to 8 years grade school, 1 to 3 years high school, 4 years high school, 1 to 3 years college, 4 years university, 5+ years university). These detailed data cover 1961 to 2001 and allow us to estimate the quality of labour input for the private business sector as well as for individual industries down to the 3-digit industry level.

The CPA are also responsible for developing internally consistent, coherent estimates of capital services. Here, the CPA rely on investment data first from the Income and Expenditures Accounts for final demand GDP and then from Input-Output Accounts that are built from industry survey data obtained from the Investment and Capital Stock Division. Investment expenditures are acquired from an establishment survey that provides industry-level data—that, at the total economy level, are reconciled with the commodity information on investment that is collected for the expenditure accounts.

At the business-sector level, the CPA employ investment series by asset classes that are available from the expenditure side of the CSNA; productivity estimates for major industry groupings are based on investment series by industry and asset classes that are included with the production accounts of the input/output tables. Because of the consistency checks that are present within the National Accounts, the investment estimates obtained from commodity data for final demand are basically the same as those derived from the production accounts.

The CPA measure the flow of capital services at the aggregate business-sector level and at the industry level. As with the estimates of labour, the CPA first develop overall capital input data that treat all assets similarly, but then account for differences among assets with different marginal products.[16]

To do so, the CPA construct detailed historical data on stock of assets by industry and asset type using investment series and estimates of depreciation (see Statistics Canada, Depreciation Rates for the Productivity Accounts, 2007). On the price side, the CPA estimate the rental price for detailed types of capital assets. We have adopted the user cost formula M1 in Table 1. That is, we have used the endogenous rate of return specification that adds in capital gains from changes in asset prices and accounts for the effects of corporate taxes. The U.S. Bureau of Labor Statistics has adopted the same method in their estimation of capital services and multifactor productivity growth.

We construct capital inputs at the industry level using as much asset detail as possible. The CPA begin with investment data for various asset types in different industries. These data come from the input-output accounts that list the investment types by industry—based on information that is attained by an industry survey of investment that is done by the Investment and Capital Stock Division. The data that are obtained by this survey are reconciled at the aggregate level with the commodity data that go into the final demand of the expenditures estimate of GDP. But here the CPA perform time-series consistency checks to reduce inconsistencies—once again employing the same criteria used in the case of labour estimates.

Once the investment series are developed that are consistent with the National Accounts, we develop a series for capital stock. To do so, we assume that there is perfect substitutability

16. See Harchaoui and Tarkhani (2003) for methodology.

among the services of different vintages τ of capital and write capital stock at time t as the sum of these capital services

(19) $$K_t = \sum_{\tau=0}^{\infty} K_{t,\tau} .$$

Under the additional assumption that the services provided by each capital good are proportional to initial investment in this good, we can express capital stock in the form

(20) $$K_t = \sum_{\tau=0}^{\infty} d_\tau I_{t-\tau} .$$

The flow of capital services is a weighted sum of past investments with weights given by the relative efficiencies $\{d_\tau\}$ of capital goods of different ages.

Capital goods decline in efficiency at each point of time, giving rise to a need for replacement if productive capacity is to be maintained. Assuming a geometric declining pattern for d_τ, we obtain

(21) $$K_t = K_{t-1}(1-\delta) + I_t = \sum_{\tau=0}^{\infty} (1-\delta)^\tau I_{t-\tau} ,$$

which is the perpetual inventory formulation of capital stock. In equation (21), δ is the geometric rate at which the efficiency of an asset declines over time.

A broad definition of capital is employed, which includes tangible assets such as equipment and structures, as well as land, and inventories. We use 28 private assets (18 types of equipment and software, 6 types of non-residential structures, and 4 types of residential structures) for about 100 industries. Capital stocks were estimated using the perpetual inventory method and a geometric depreciation rate based on age-price profiles developed by Statistics Canada, Depreciation Rates for the Productivity Accounts (2007). Important exceptions are the depreciation rates for assets in the structures category. Owing to a lack of an active transaction market for structures, depreciation rates were derived from the existing information on length of lives from a survey done by the Investment and Capital Stock Division that produces expected length of life by asset type.

We also assemble data on investment and land to complete our capital estimates. Inventories and land are assumed to have a depreciation rate of zero and do not face an investment tax credit or capital consumption allowance, so the rental price formula is a simplified version of the one employed for reproducible assets.

These stocks are then aggregated into a capital input measure using a Fisher index. The rental prices are used as weights for the aggregation of the various asset-type capital stocks. This approach, originated by Jorgenson and Griliches (1967), is based on the identification of rental prices with marginal revenue products of different types of capital. Our estimates of these prices incorporate differences in asset prices, service lives and depreciation rates, and the tax treatment of capital incomes.

Table 2 Depreciation rate by asset type

Asset type	Geometric depreciation rate
Office furniture, furnishing	0.24
Non-office furniture, furnishings and fixtures	0.21
Motors, generators, and transformers	0.13
Computer-assisted process	0.17
Non-computer-assisted process	0.16
Communication equipment	0.22
Tractors and heavy construction equipment	0.17
Computers, associated hardware and word processors	0.47
Trucks, vans, truck tractors, truck trailers and major replacement parts	0.23
Automobiles and major replacement parts	0.28
Other machinery and equipment	0.20
Electrical equipment and scientific devices	0.22
Other transportation equipment	0.10
Pollution abatement and control equipment	0.15
Software	0.55
Plants for manufacturing	0.09
Farm buildings, maintenance garages, and warehouses	0.08
Office buildings	0.06
Shopping centers and accommodations	0.07
Passenger terminals, warehouses	0.07
Other buildings	0.06
Institutional building construction	0.06
Transportation engineering construction	0.07
Electric power engineering construction	0.06
Communication engineering construction	0.12
Downstream oil and gas engineering facilities	0.07
Upstream oil and gas engineering facilities	0.13
Other engineering construction	0.08
Land	0.00
Inventories	0.00

Source: Statistics Canada, Depreciation Rates for the Productivity Accounts (2007).

It should be noted that the methodology and the data used for the experiments reported in this paper differ from those used in the estimates published by Statistics Canada. First, they make use of preliminary data that are reclassified from the old Standard Industrial Classification (SIC) system to the new North American Industrial Classification System (NAICS). Second, we have, for the sake of simplicity, used average asset prices rather than industry-specific prices. And for our estimates of cost of capital for the aggregate business sector calculations, we have averaged the return across all industries. Finally, depreciation rates are taken from a new set of estimates that are in the process of being developed from a new database that yields slightly lower estimates than those used presently. Table 2 contains depreciation rates for 30 assets that will be used for this paper.

Table 3 A comparison of alternative capital rental cost formulae in the business sector, 1961 to 1981

	Endogenous rate of return			Exogenous rate of return		
	M1	M2	M3	M4	M5	M6
Mean statistics over years						
Average nominal rate of return	0.14	0.14	0.12	0.11	0.11	0.11
Average cost share of capital	0.38	0.38	0.38	0.31	0.32	0.35
Annual change in capital rental price (%)	5.48	5.52	5.77	5.49	6.40	6.17
Annual growth in capital quality (%)	0.81	0.77	0.52	1.02	0.95	0.59
Annual multifactor productivity growth (%)	0.90	0.91	1.01	1.26	1.23	1.18
Standard deviation over years						
Average nominal rate of return	0.049	0.040	0.035	0.034	0.034	0.034
Average cost share of capital	0.012	0.012	0.012	0.065	0.043	0.017
Annual change in capital rental price (%)	0.048	0.048	0.049	0.260	0.078	0.034
Annual growth in capital quality (%)	0.006	0.005	0.004	0.007	0.006	0.004
Annual multifactor productivity growth (%)	1.868	1.848	1.806	1.712	1.816	1.778
Percent of negative rental price	0.33	0.00	0.00	1.33	0.83	0.00

Source: Statistics Canada, Canadian Productivity Accounts.

5. *Empirical results*

In this paper, we focus on the productivity of the aggregate business sector. In order to estimate alternate productivity estimates in this paper, we make use of an aggregate production function approach. To do so, we treat aggregate output as the sum of gross domestic product (GDP) of industries that belong to the business sector and use aggregate labour and aggregate capital input. The aggregate capital input is calculated as the aggregation of capital stock of all types of reproducible fixed assets, inventories and land in the business sector. The aggregate labour input is the aggregation of workers with different education attainment and different experience.

5.1 *Results for the aggregate business sector*

In order to assess the effect of the six alternate scenarios, we compare the average rates of return that are produced by each, the share accruing to capital, the increase in the price of capital services, the increase in capital composition (the difference between the growth of capital services and the growth in capital stock), and finally the growth in multifactor productivity (MFP). The six alternative user-cost formulae all incorporate the effects of corporate tax provisions. They differ by the choice of the rate of return and the choice of expected capital gains. Summary statistics in each of these areas can be found for the period 1961 to 1981 in Table 3 and for 1981 to 2001 in Table 4. Figures 1 to 6 summarize differences across estimates M1 to M6 for the entire time period.

Table 4 A comparison of alternative capital rental cost formulae in the business sector, 1981 to 2001

	Endogenous rate of return			Exogenous rate of return		
	M1	M2	M3	M4	M5	M6
Mean statistics over years						
Average nominal rate of return	0.09	0.09	0.10	0.09	0.09	0.09
Average cost share of capital	0.40	0.40	0.40	0.41	0.40	0.38
Annual change in capital rental price (%)	2.81	2.84	3.01	2.24	1.57	2.02
Annual growth in capital quality (%)	1.09	1.06	0.89	1.09	1.08	0.94
Annual multifactor productivity growth (%)	0.30	0.31	0.38	0.32	0.33	0.43
Standard deviation over years						
Average nominal rate of return	0.026	0.020	0.024	0.025	0.025	0.025
Average cost share of capital	0.018	0.018	0.018	0.072	0.053	0.022
Annual change in capital rental price (%)	0.057	0.057	0.057	0.126	0.061	0.028
Annual growth in capital quality (%)	0.004	0.004	0.003	0.004	0.004	0.003
Annual multifactor productivity growth (%)	1.654	1.663	1.687	1.697	1.718	1.714
Percent of negative user cost	0.00	0.00	0.00	0.00	0.00	0.00

Source: Statistics Canada, Canadian Productivity Accounts.

5.1.1 Rates of return

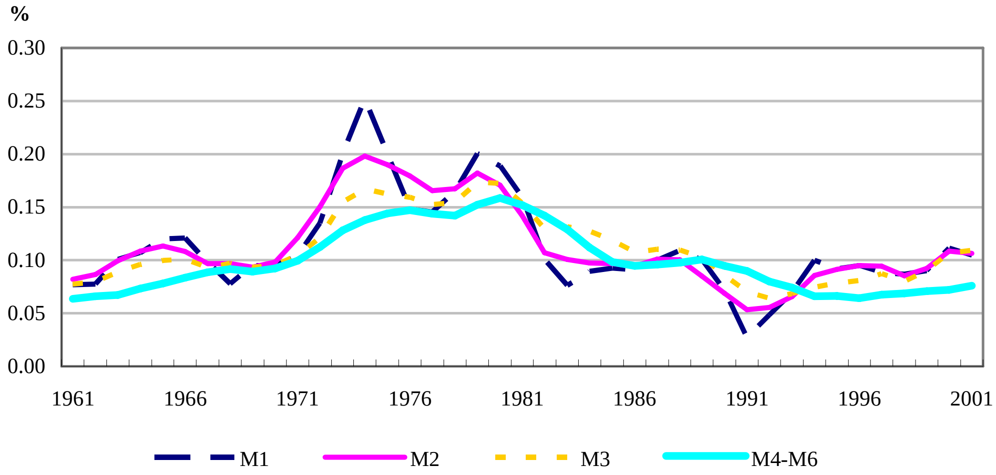
The nominal rates of return that are produced by the endogenous method are generally higher than those for the exogenous method.[17] Over the 1961 to 1981 period, the endogenous rate that excludes asset price changes as a measure of capital gains (M3) is 12%, while the comparable exogenous rate averages only 11% (Table 3). Over the 1981 to 2001 period, the endogenous rate estimated from M3 is 10% while the comparable exogenous rate averages 9% (Table 4). This difference could arise either because our exogenous rates are calculated using too few asset types, or because they contain a residual that is really due to scale economies. But it should be noted that the difference is not large—around 1 percentage point in both periods when we consider M3 and M6, the two methods that exclude asset price changes as measures of capital gains.

The trend of the two series that excludes asset price changes as a measure for capital gains (M3 and M6) is quite similar (Figure 1). Both increase during the mid 1970s as a result of an increase in inflation and then decrease in the 1980s. Since the exogenous rate consists of a constant real return and the consumer price index, the similarity in the rate of return between M3 and M6 shows that the endogenous real user cost of capital is relatively constant as well.

But there are very different movements in the series of rates of return that are calculated net and gross of capital gains. Consider the endogenous options that include capital gains (M1 and M2). The endogenous after-tax return gross that was calculated using instantaneous asset price changes to capture capital gains (M1) is more volatile than the rate that is inclusive of asset price changes smoothed with a 5-year moving average (M2) and the rate that does not have added into

17. We have employed the user cost formula based on real rates in estimating the exogenous user cost of capital. For the presentation of the results, we will use nominal rates of return. The nominal rates of return are computed as the sum of the real rates of return plus a 5-year moving average of change of the consumer price index.

Figure 1 Nominal after tax rate of return in the business sector



Source: Statistics Canada, Canadian Productivity Accounts.

it, the changes in asset prices (M3). The volatility of the latter two rates (the rate inclusive of smoothed capital gains and the rate without asset price changes) is similar.

There are also different movements in the series of the endogenous and exogenous rates of return. The endogenous rate of return that is calculated using both instantaneous and smoothed asset price changes is much more volatile than the exogenous rate of return. But the difference in the volatility between the endogenous rate gross of smoothed capital gains, the endogenous rate net of capital gains and the exogenous rate is quite small.

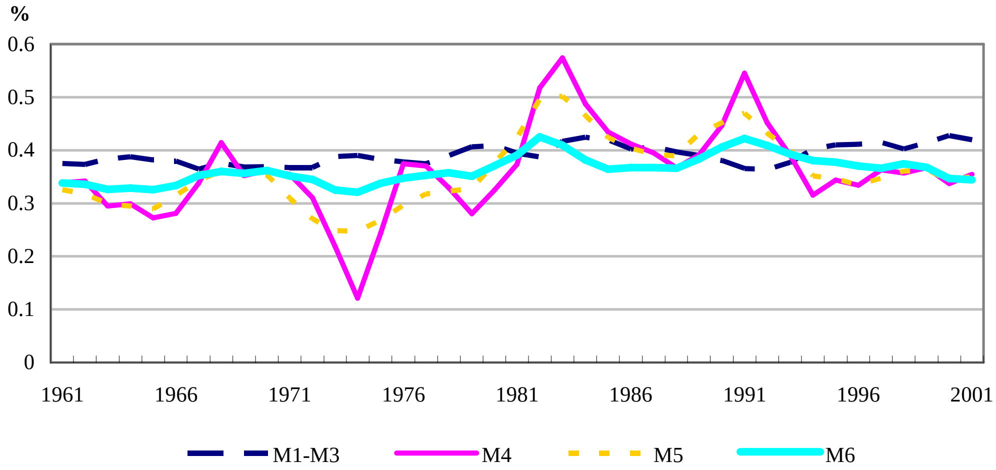
5.1.2 Cost share of capital

The formula for estimating multifactor productivity (7) includes a term representing the share of capital. In a world where there is only one asset type, the share of capital is the marginal revenue product of capital multiplied by the capital stock of the asset divided by the value of total output.

For the endogenous rate of return, this term is derived from the share of national income going to capital. In the exogenous alternative, the share can no longer be presumed to exhaust surplus and has to be estimated directly from the product of the cost of capital and the capital stock of the asset. The cost share may not be the same as the surplus—either because the rates of return chosen underestimate the true returns or because not all assets (for example, intangible assets) are included. In this paper, we recalculate the cost shares for the exogenous rate using the user cost of capital relevant to the method concerned.

The capital shares for the endogenous methods (M1 to M3) are 3 percentage points higher than for the exogenous alternative that omits capital gains (M6) (Tables 3 and 4). And the time trends of these series are broadly similar—though the share derived from the endogenous rate tends to increase in the latter part of the period while the exogenous rate declines. The latter suggests that if intangible and other assets associated with the knowledge economy account for the differences in the two methodologies, they have become more important in more recent years.

Figure 2 Cost share of capital in the business sector



Source: Statistics Canada, Canadian Productivity Accounts.

The cost share of capital is more stable for the endogenous rate specifications than for the exogenous rate specifications. In particular, the cost share of capital for the endogenous rate specification with annual asset price change being used to represent capital gains (M1) is more stable than the cost share of capital for the exogenous rate specification. This occurs despite the fact that the former specification has the most volatile rate of return and the most volatile capital gains. The volatility of the rate of return and capital gain series for M1 is offsetting, which yield a more stable cost-share series.

The stability of the cost-share series arising from the endogenous alternative is an advantage of this technique if a proxy is desired to track the time path of the marginal product of capital that is regarded as not being highly volatile.

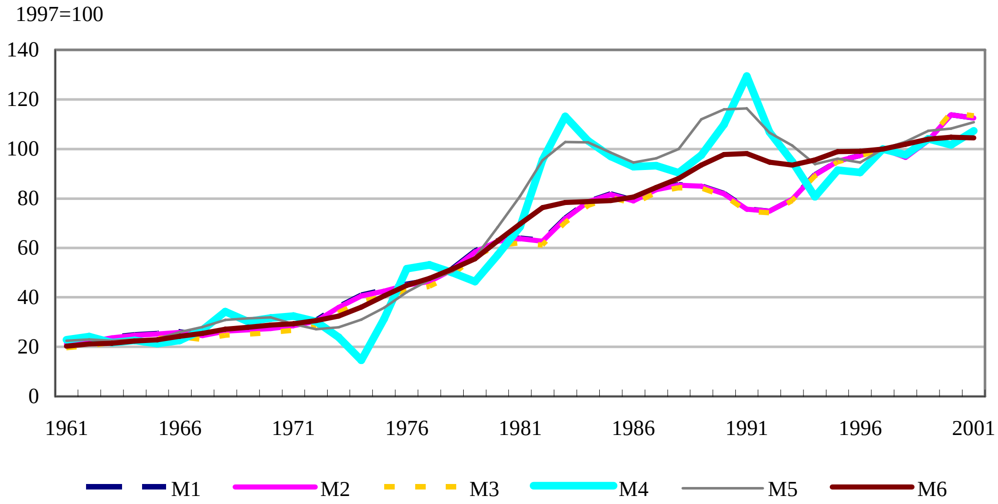
There is a considerable difference in the time pattern of the cost share of the two exogenous rates that take into account asset price changes (M4 and M5). This is because of the volatility in the series of asset price changes that cause the user cost of capital to fluctuate. The cost share of capital for the exogenous rate method that accounts for capital gains that is an average of 5-year asset price changes is more stable than the cost share of capital for the endogenous rate method that accounts for annual capital gains.

The cost share of the exogenous rate that omits capital gains as measured by asset price changes is less volatile than the cost share of the other two exogenous rates that account for capital gains. That is because the user cost derived from the former method is essentially free of the asset price changes that are a source of fluctuation in the cost share of capital.

5.1.3 Cost of capital services

The growth in the rental price of capital over time reflects changes in the price of capital goods, the rate of return, the depreciation rate and capital gains. The annual growth rate is lower for the endogenous than for the exogenous rate in the period 1961 to 1981 and higher in the period 1981

Figure 3 The cost of capital services in the business sector (1997=100)



Source: Statistics Canada, Canadian Productivity Accounts.

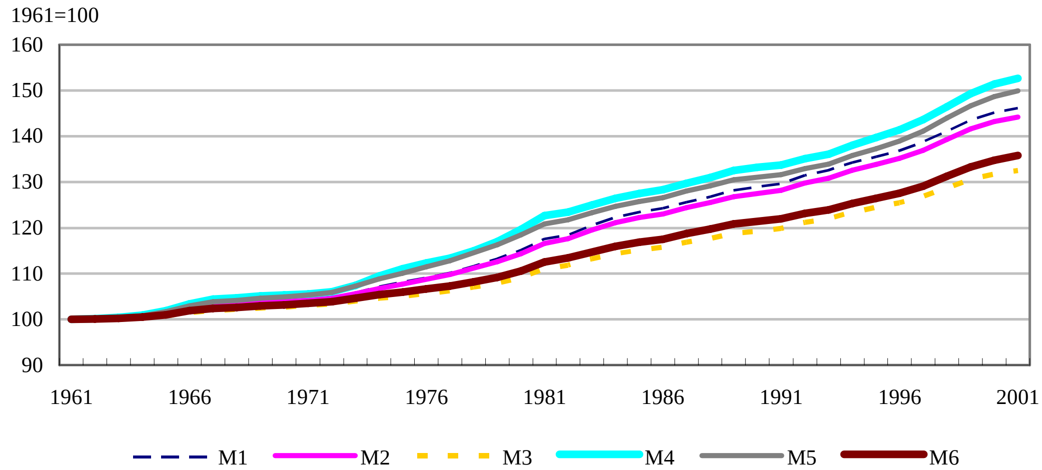
to 2001. The difference occurs primarily because capital gains, when measured by asset price changes, are positive for the first period and negative for the latter. Over the entire period, there is little difference in the cumulative growth of each method.

Volatility in the user cost of capital calculated using the exogenous rate is quite different from the volatility in the user cost calculated using the endogenous rate. The normal procedure (Harper, Berndt and Wood, 1989; and Schreyer, Bignon and Dupont, 2003) is to presume that the exogenous rate chosen includes capital gains and thus subtracts gains in asset prices from the exogenous rate of return. This causes the user cost of capital to be highly volatile for the exogenous rate method—in contrast with the result obtained from the endogenous procedure where it is relatively constant in nominal terms and even steadier when calculated in real terms. The evidence in Tables 3 and 4 shows that the user cost of capital is most volatile for the exogenous rate specification that accounts for annual capital gains. It is only when asset price changes are excluded that the exogenous rate method yields a stable series of the cost of capital. Once more, the lower volatility of the endogenous rate favours its use by those who believe that the marginal product of capital changes only slowly over time.

It should be noted that the endogenous and the exogenous methods yield quite different cyclical movements. The endogenous rental price of capital moves in a procyclical way, declining in recessions in the early 1980s and 1990s. This occurs because the endogenous rate of return has a procyclical tendency while the exogenous rate by definition does not. Procyclicality may be advantageous because it allows a multifactor productivity measure to partially take into account that considerable amounts of capital are not being employed in recession.[18]

18. For more discussion of these issues, see Berndt and Fuss (1986), and Hulten (1986).

Figure 4 Indices of capital composition in the business sector, 1961 to 2001



Source: Statistics Canada, Canadian Productivity Accounts.

5.1.4 Indices of capital composition

Capital composition is the ratio of the rental price-weighted aggregate capital service flows of capital assets to the unweighted aggregate capital service flow. Capital composition will be positive if the fastest growing assets have a higher weight—if their user cost of capital is higher.

No matter what technique is used to approximate capital gains, the growth in capital composition is lower for the endogenous method than it is for the exogenous method. Schreyer (2004) reports similar findings for a number of the Organisation for Economic Co-operation and Development (OECD) countries.

Moving from an endogenous rate of return to an exogenous rate of return leads to an increase in the growth of capital composition and capital services. This result is due to three factors. First, the estimated endogenous rate of return is higher than the exogenous rate of return in the Canadian business sector. Second, a higher rate of return leads to a lower growth of capital composition and capital service input, as the use of higher rate of return in the user cost calculation attenuates the difference in the user cost of capital between assets arising from differences in depreciation rates. Third, there is a positive correlation between depreciation and the rate of growth of capital. Adding in a larger but constant rate of return to the depreciation rate attenuates this correlation and thereby results in a lower growth of capital composition and capital services.

It is also the case that the growth in capital composition is lower for the case when capital gains, derived from asset price changes, are not included in the estimation of user cost. The capital rental cost formulae that incorporate the asset-specific capital gains yields a higher capital composition effect than the capital rental cost formulae that do not. This result is due to (1) the long-run historical shift toward equipment (with relatively high depreciation and high user cost) and away from structures (with relatively low depreciation and low user cost), which increases the capital composition effect; and (2) the long-run tendencies for the price of structures (with low depreciation) to increase faster than the price of equipment (with high depreciation), causing

Table 5 Correlation coefficients between annual capital stock growth, user cost, annual capital gains and depreciation

	Capital stock growth	User cost	Capital gains	Depreciation rate
1961 to 2001				
Capital stock growth	1.000	…	…	…
User cost	0.132	1.000	…	…
Capital gains	0.004	-0.326	1.000	
Depreciation rate	0.217	0.462	-0.180	1.000
1961 to 1981				
Capital stock growth	1.000	…	…	…
User cost	0.070	1.000	…	…
Capital gains	-0.066	-0.048	1.000	
Depreciation rate	0.139	0.182	-0.165	1.000
1981 to 2001				
Capital stock growth	1.000	…	…	…
User cost	0.401	1.000	…	…
Capital gains	-0.193	-0.350	1.000	
Depreciation rate	0.359	0.617	-0.170	1.000

… not applicable

Notes: Correlation coefficients are calculated using asset-specific capital stock as weights. The user cost of capital is estimated using M1.

Source: Statistics Canada, Canadian Productivity Accounts.

the capital gains that are subtracted in the user cost of structures formulae to be larger than those subtracted in the user cost of equipment estimates. This increases the difference in the user cost of structures and equipment, and thus leads to an increase in the growth in capital composition.

To better understand the effect of capital gains on growth in capital composition effect, we have estimated the correlation coefficients between capital stock growth, depreciation, and investment price changes across assets in Table 5. We find that there is a positive correlation between depreciation and capital stock growth across assets, which reflects the long-run historical shift toward equipment (with relatively high depreciation and high user cost) and away from structures (with relatively low depreciation and low user cost). We also find that there is a negative correlation between depreciation and capital gains, which reflects the long-run tendencies for the price of structures (with low depreciation) to increase faster than the price of equipment (with high depreciation).

There is no apparent ranking of the endogenous and exogenous rate of return methods in terms of the volatility of the capital composition series. For the 1961 to 1981 period, the endogenous rate method yields a more stable capital composition series, while for the 1981 to 2001 period, the endogenous and exogenous rate methods yield capital composition series that have similar volatility.

Table 6 The effect of changing nominal rate of return on capital composition growth and multifactor productivity growth

	Exogenous rate of return times				
	0.50	0.75	1.00	1.25	1.50
Annual capital quality growth					
Period 1961 to 1981	1.52	1.20	0.95	0.75	0.60
Period 1981 to 2001	1.54	1.27	1.08	0.93	0.82
Average cost share of capital					
Period 1961 to 1981	0.17	0.25	0.32	0.39	0.46
Period 1981 to 2001	0.28	0.34	0.40	0.46	0.53
Annual multifactor productivity growth (%)					
Period 1961 to 1981	1.97	1.58	1.23	0.90	0.58
Period 1981 to 2001	0.56	0.44	0.33	0.21	0.10

Note: Based on M5: exogenous rate of return with smoothed capital gains.
Source: Statistics Canada, Canadian Productivity Accounts.

5.1.5 Indices of multifactor productivity growth

Differences in multifactor productivity (MFP) growth arise from differences in the cost shares being used and differences in the growth in capital composition across methods.

Multifactor productivity growth is faster when we use the exogenous rate of return rather than endogenous rate of return. Schreyer (2004) reports similar findings for a number of OECD countries.

This difference arises because the exogenous rate of return is lower than the endogenous rate of return. The level of the nominal rate of return affects MFP growth via its effects on capital composition and the cost share of capital. The use of a lower rate in the user cost estimation leads to higher growth of capital composition and lower cost share of capital service in the MFP growth accounting. The former leads to a decline in the MFP growth estimate while the latter leads to an increase in the MFP growth estimate. The overall effect of the two offsetting factors is an increase in the MFP growth rate as the effect of changes in capital share tends to dominate the effect of changes in capital composition.

To further examine the effect of changing the nominal rate of return on capital composition growth and MFP growth, we have estimated capital composition growth and MFP growth using different nominal rates of return (Table 6). We find that increases in the nominal rate of return lead to decreases in capital composition growth, increases in cost share of capital, and decreases in MFP growth. These findings underscore the importance of choosing the appropriate exogenous rate of return in order to avoid errors in the estimation of MFP. Moreover, they give us some indication of the type of confidence interval that we should use to guide the user community regarding the accuracy of MFP statistics. Using Table 6, we calculate that a deviation of the rate of return of 25% around the true value (which would correspond to a range of 3% to 5% around the value of 4% in the value of the rate of return) produces a range of around ± 0.34 percentage point or ± 28% for the 1.23% multifactor productivity growth rate derived for the period from 1961 to 1981 and ± 0.11 percentage point or ± 33% for the period from 1981 to 2001.

The estimated MFP growth is lower for the user cost method that accounts for capital gains with asset price changes than the comparable method that does not. This is a result of the higher capital composition effect for the estimate that accounts for capital gains. But if the capital gains term is added and not subtracted (M5*),[19] the annual growth in MFP falls from 1.23% to 1.03% during the period from 1961 to 1981.

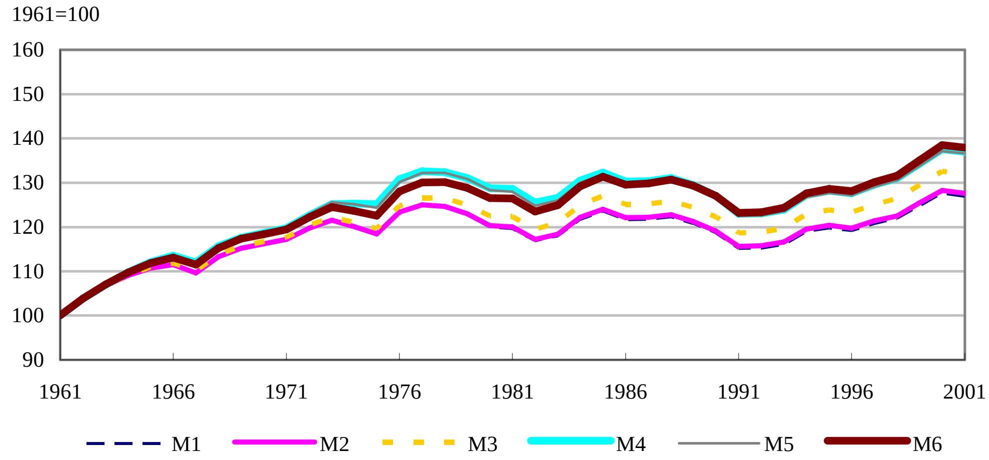
The use of a longer run (e.g., a 5-year centered moving average) versus annual capital gains has little effect on the MFP growth and capital composition. But it has an effect on the volatility of the capital rental price and the rate of return. The estimated rate of return is most volatile when we use annual realized capital gains from asset price changes rather than an estimate of long-run capital gains. The estimated capital rental price is most volatile when we use realized capital gains together with an exogenous rate of return. Once again, this suggests that the exogenous rate is less suitable if the marginal product of capital remains relatively stable over time.

Our findings on the effects of including capital gains as instantaneous price changes or by averaging them over time speaks to the measurement debate between Denison (1969), and Jorgenson and Griliches (1967). Jorgenson and Griliches used an annual adjustment for capital gains (M1). Denison argued that incorporating a long-run average of capital gains may be more appropriate (M2). Our findings for Canada show that the choice of realized capital gains versus long-run average capital gains has little effect on the mean and the variance of annual MFP growth and annual capital composition growth. The main effect is on the volatility in the internal rate of return and the rental price of capital.

Finally, it should be noted that the differences in alternate methods are most relevant to inferences that are made about long-run trends in multifactor productivity. Figure 6 depicts differences across the six alternate methods for annual rates of change over the period from 1961 to 2001. It is difficult to distinguish one method from another in the annual data. It is only in Figure 5 that differences in cumulative effects emerge over longer periods of time.

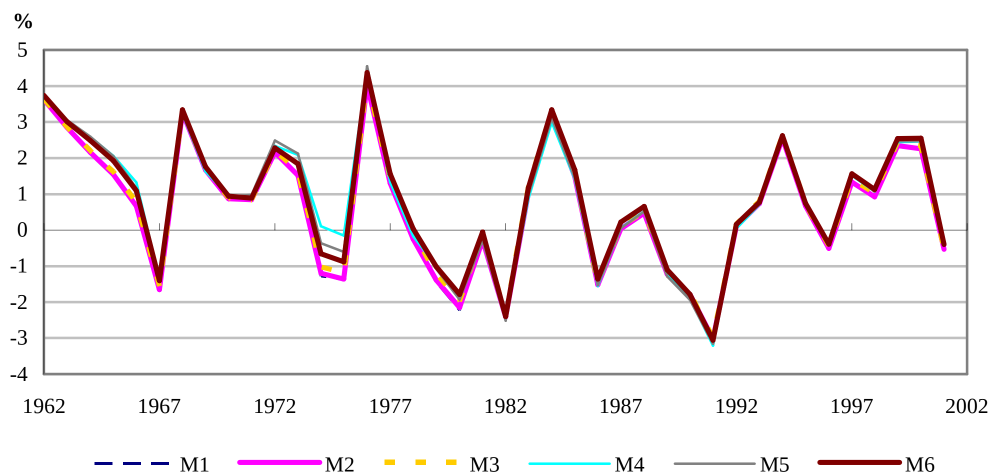
19. This is the appropriate strategy if the exogenous rate is calculated net of capital gains, which would be the case if the equity rate embedded in the exogenous rate reflected capital gains accruing to equity owners.

Figure 5 Multifactor productivity in the business sector, 1961 to 2001



Source: Statistics Canada, Canadian Productivity Accounts.

Figure 6 Multifactor productivity growth in the business sector



Source: Statistics Canada, Canadian Productivity Accounts.

5.2 The biases of ignoring corporate tax provisions

Tax considerations are a key component of capital services prices. However, empirical studies often fail to incorporate the effects of taxes in the user cost calculation. In this section, we examine the bias in our estimates as a result.

The same endogenous and exogenous methods were used to recalculate the unit price of capital before tax and the other the relevant variables used to estimate multifactor productivity (MFP). The user cost formula that does not account for the effects of taxes is written in (12). In the formula, r_t is the rate of return that is the gross of corporate taxes. Therefore, the rate of return for the exogenous rate method should be chosen as such. The return to debt or a weighted sum of

Table 7 Biases of ignoring corporate taxes in the user cost estimation for the business sector, 1961 to 1981

	Endogenous rate of return			Exogenous rate of return		
	M7	M8	M9	M10	M11	M12
Average nominal rate of return	0.20	0.20	0.19	0.19	0.19	0.19
Average cost share of capital	0.38	0.38	0.38	0.38	0.38	0.40
Annual change in capital rental price (%)	5.48	5.52	5.70	7.18	7.78	7.62
Annual growth in capital quality (%)	0.81	0.77	0.59	0.81	0.75	0.55
Annual multifactor productivity growth (%)	0.90	0.91	0.98	0.94	0.94	0.91
Percent of negative user cost	0.17	0.00	0.00	0.50	0.00	0.00
	Bias of ignoring tax provisions					
Average nominal rate of return	0.06	0.06	0.06	0.08	0.08	0.08
Average cost share of capital	0.00	0.00	0.00	0.06	0.06	0.05
Annual change in capital rental price (%)	0.00	0.00	-0.07	1.68	1.38	1.45
Annual growth in capital quality (%)	0.00	0.00	0.07	-0.21	-0.20	-0.04
Annual multifactor productivity growth (%)	0.00	0.00	-0.03	-0.32	-0.30	-0.26

Note: The bias of ignoring taxes in the user cost estimation is the difference between the user cost estimation that does not account for taxes and the user cost estimation that accounts for taxes.
Source: Statistics Canada, Canadian Productivity Accounts.

the returns to debt and equity is often used for the exogenous rate method. But those rates are often rates that are net of corporate taxes and they should be converted to before-tax rates. In this paper, we calculate the before-tax rate of return for the exogenous method as the after-tax rate of return divided by $(1-u)$, where u is the statutory tax rate and the after-tax rate of return is a weighted sum of returns to corporate equity and debt.

Tax considerations affect growth in the estimate of the impact of changes in capital composition through their effect on the relative user cost of different assets. On one hand, the inclusion of tax parameters reduces the rate of return. Our results suggest that this will accentuate the difference between high user cost and low user cost of capital assets, thus leading to higher capital composition effect (Table 6). On the other hand, the inclusion of tax parameters reduces the user cost of relatively high depreciating and high rental-price assets as a larger capital consumption allowance term is subtracted from the user costs of those assets. This will attenuate the user cost difference between assets and lead to a lower capital composition effect. These factors offset one another.

Table 8 Biases of ignoring corporate taxes in the user cost estimation for the business sector, 1981 to 2001

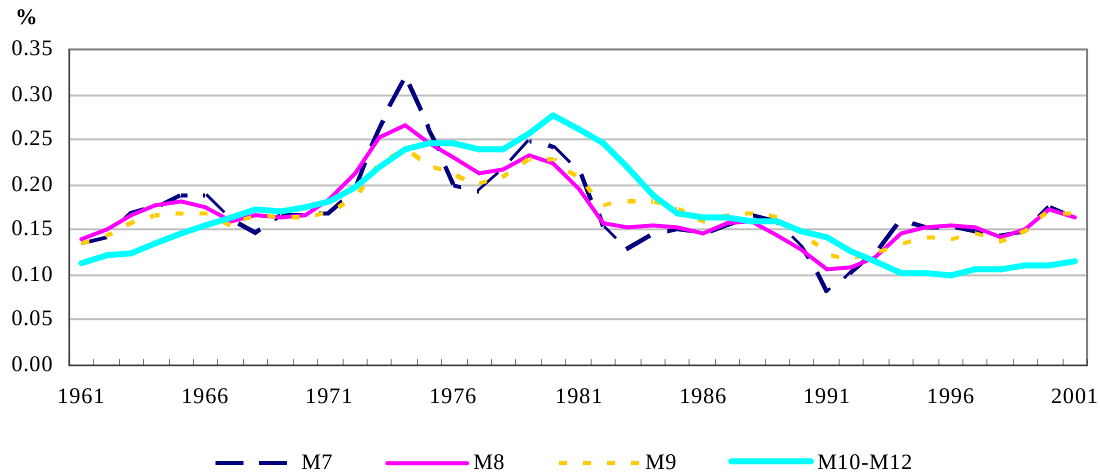
	Endogenous rate of return			Exogenous rate of return		
	M7	M8	M9	M10	M11	M12
Average nominal rate of return	0.15	0.15	0.16	0.15	0.15	0.15
Average cost share of capital	0.40	0.40	0.40	0.41	0.41	0.39
Annual change in capital rental price (%)	2.81	2.83	2.98	0.57	0.16	0.46
Annual growth in capital quality (%)	1.08	1.07	0.92	1.13	1.11	0.97
Annual multifactor productivity growth (%)	0.30	0.30	0.36	0.31	0.32	0.41
Percent of negative user cost	0.00	0.00	0.00	0.00	0.00	0.00
	Bias of ignoring tax provisions					
Average nominal rate of return	0.06	0.06	0.06	0.06	0.06	0.06
Average cost share of capital	0.00	0.00	0.00	0.00	0.00	0.01
Annual change in capital rental price (%)	0.00	-0.01	-0.02	-1.67	-1.41	-1.56
Annual growth in capital quality (%)	0.00	0.01	0.02	0.04	0.04	0.03
Annual multifactor productivity growth (%)	0.00	0.00	-0.01	0.00	-0.01	-0.02

Note: The bias of ignoring taxes in the user cost estimation is the difference between the user cost estimation that does not account for taxes and the user cost estimation that accounts for taxes.
Source: Statistics Canada, Canadian Productivity Accounts.

The results are presented in Tables 7 and 8 for the period 1961 to 1981 and 1981 to 2001 respectively. Figures 7 to 11 summarize the results using different methods for each year of the period 1961 to 2001. We find that ignoring corporate taxes leads to an upward bias in the estimated capital composition effect for the endogenous rate method for both the 1961 to 1981 and 1981 to 2001 periods. For the exogenous rate methods, it leads to an upward bias in capital composition in the 1981 to 2001 period, and a downward bias in capital composition in the 1961 to 1981 period.

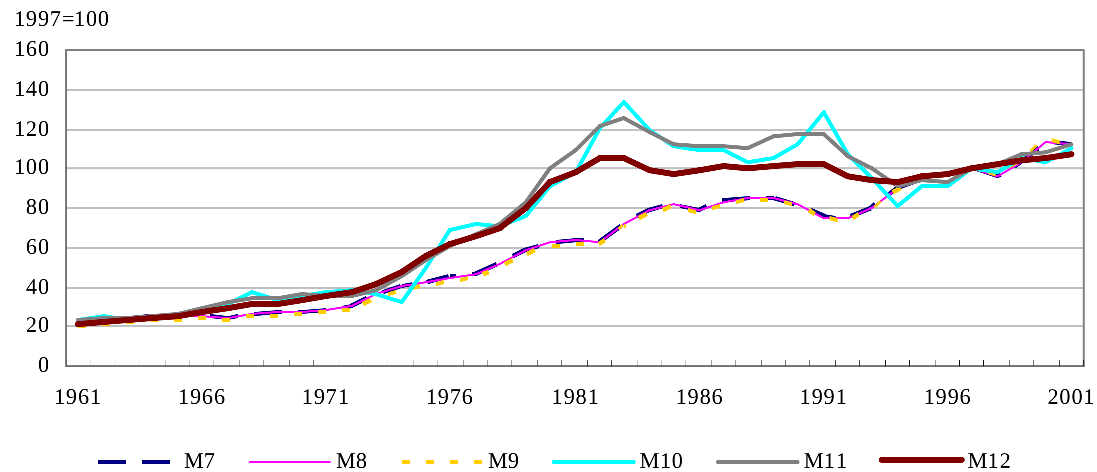
The user cost estimation that does not take into account tax considerations leads to a downward bias in estimated MFP growth for the endogenous rate methods. This is a result of the upward bias in the estimated capital composition effect for those methods. For the exogenous rate methods, ignoring taxes introduces a bias in the cost share of capital that is in addition to the bias in the capital composition effect. This additional bias associated with the exogenous rate method is significant and leads to a much larger bias in the estimated MFP growth for the 1961 to 1981 period. For the 1981 to 2001 period, the additional bias in the cost share of capital is small and thus has little effect on the estimated MFP growth.

Figure 7 Nominal before-tax rate of return in the business sector



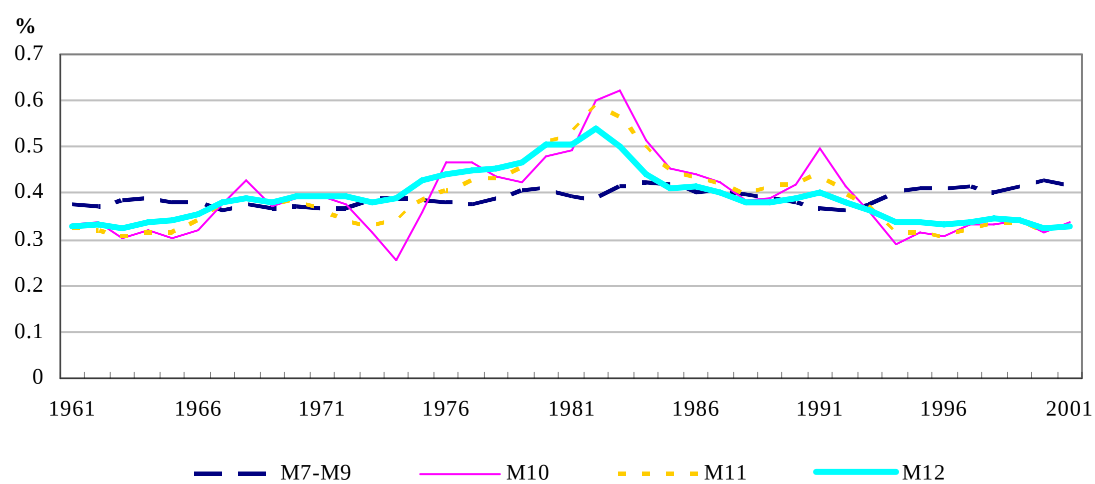
Source: Statistics Canada, Canadian Productivity Accounts.

Figure 8 The cost of capital services in the business sector, no tax considerations



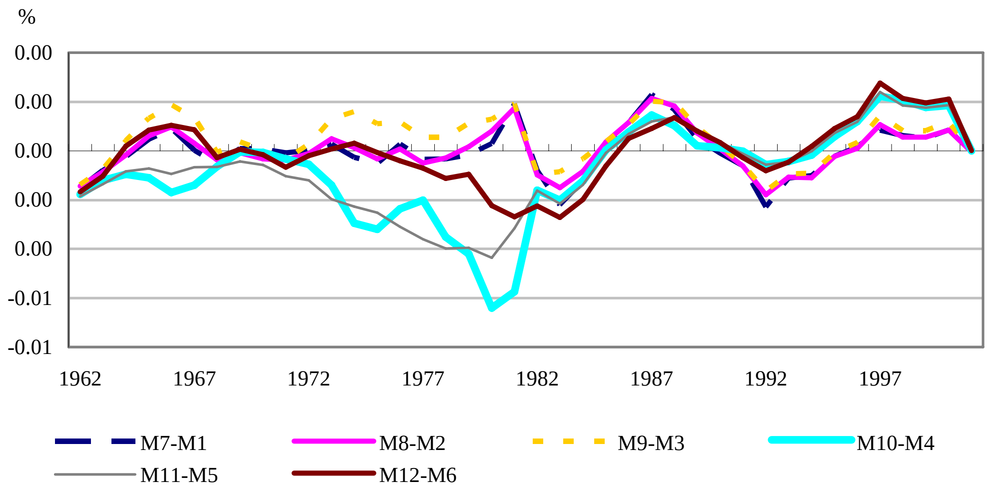
Note: 1997=100.
Source: Statistics Canada, Canadian Productivity Accounts.

Figure 9 Cost share of capital in the business sector, no tax considerations



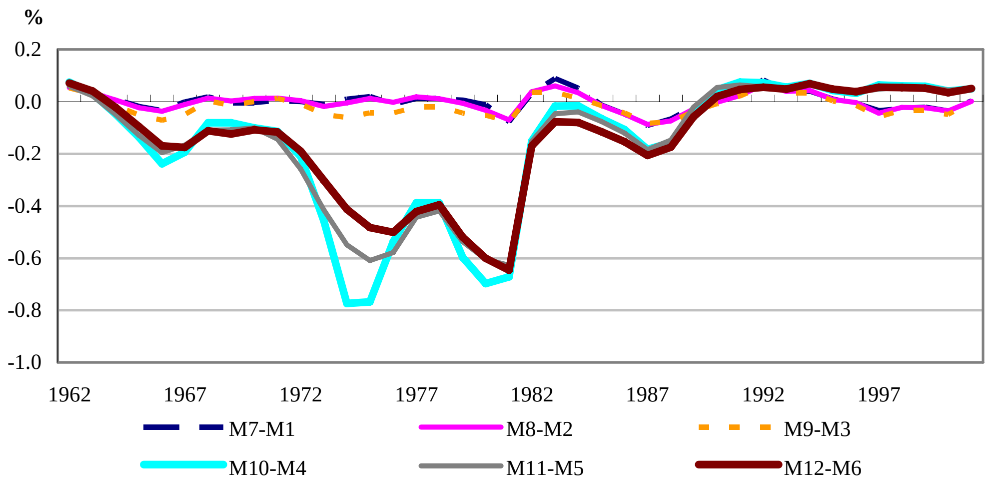
Source: Statistics Canada, Canadian Productivity Accounts.

Figure 10 Effects of ignoring tax provisions in user-cost formulae on capital quality growth of the business sector



Source: Statistics Canada, Canadian Productivity Accounts.

Figure 11 Effects of ignoring tax provisions in user-cost formulae on multifactor productivity growth of the business sector



Source: Statistics Canada, Canadian Productivity Accounts.

Figures 10 and 11 show that the bias of ignoring taxes in the annual MFP and capital composition estimates is much larger for the exogenous rate method than for the endogenous rate method. Once more, this suggests that the endogenous method has an advantage over the exogenous method.

Table 9 The list of industries for the user cost estimation at the industry level

Code	Industry	Code	Industry
1	Agriculture	19	Machinery manufacturing
2	Mining	20	Computer and electronic product
3	Utilities	21	Electrical equipment
4	Construction	22	Transportation equipment
5	Food	23	Furniture
6	Beverage and tobacco	24	Miscellaneous manufacturing
7	Textile	25	Wholesale trade
8	Clothing	26	Retail trade
9	Leather and allied	27	Transportation
10	Wood product	28	Information & cultural
11	Paper manufacturing	29	Finance, insurance & real estate
12	Printing	30	Professional & technical services
13	Petroleum and coal products	31	Administrative & waste management
14	Chemical manufacturing	32	Educational services
15	Plastics and rubber	33	Health care services
16	Non-metallic mineral	34	Arts and entertainment
17	Primary metal	35	Accommodation
18	Fabricated metal product	36	Other services

Source: Statistics Canada, Canadian Productivity Accounts.

5.3 Results for the detailed industries

In this section, we will examine the effects of alternative rental price formulae on the estimated capital composition and multifactor productivity growth at a detailed industry level. For that purpose, we will use an industry KLEMS (capital, labour, energy, material and services) data set covering 36 industries of the Canadian business sector over the period from 1961 to 2001. The data set is obtained from a more detailed industry KLEMS data set covering 88 industries of the business sector based on the North American Industrial Classification System (NAICS) industry classifications. The list of the 36 industries is presented in Table 9.

Multifactor productivity (MFP) growth at the industry level is based on gross output and is constructed as the difference between gross output growth and the weighted growth of capital, labour and intermediate inputs.

The top panel of Table 10 presents a simple average of the nominal rate of return, the cost share of capital, annual capital composition growth and annual MFP growth in the period from 1981 to 2001 over 36 industries. We find that the endogenous rate of return is higher than the exogenous rate of return over the period 1981 to 2001. For that period, the average exogenous rate is 0.09, while the average endogenous rate of return is 0.21 from the rental price specifications M1 and M2 and 0.22 from the rental price specification M3.

Table 10 The effects of alternative user cost formulae on capital services and multifactor productivity growth at the industry level, 1981 to 2001

	M1	M2	M3	M4	M5	M6
Average of statistics over industries						
Nominal rate of return	0.21	0.21	0.22	0.09	0.09	0.09
Cost share of capital	0.17	0.17	0.17	0.15	0.15	0.14
Annual % change in user cost	3.01	3.04	3.21	1.73	1.26	1.71
Annual capital composition (%)	0.87	0.84	0.67	1.07	1.05	0.90
Annual multifactor productivity growth (%)	0.12	0.12	0.15	0.26	0.26	0.30
Average of standard deviation of statistics over industries						
Nominal rate of return	0.082	0.078	0.079	0.025	0.025	0.025
Cost share of capital	0.026	0.026	0.026	0.038	0.031	0.022
Annual % change in user cost	20.683	20.685	20.661	13.518	6.180	3.529
Annual capital composition (%)	1.132	1.108	0.962	1.483	1.470	1.367
Annual multifactor productivity growth (%)	2.430	2.429	2.425	2.454	2.454	2.431
Percent of negative user cost	0.77	0.77	0.70	0.00	0.00	0.00

Source: Statistics Canada, Canadian Productivity Accounts.

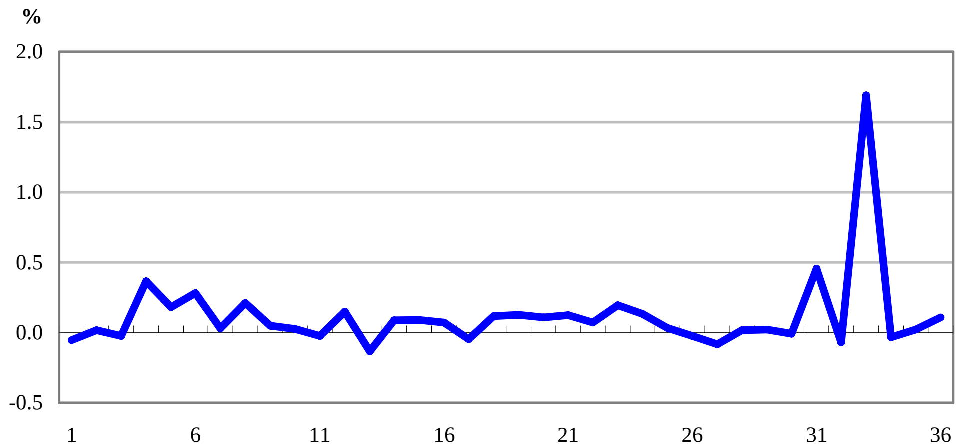
The relatively high endogenous rate of return produces an estimate of the growth in capital composition that is lower for the endogenous rate method than for the exogenous rate method and a lower MFP growth. For example, when we allow for long-run capital gains (M2), annual MFP growth is 0.12% and annual capital composition growth is 0.84% over the period 1981 to 2001 for the endogenous rate method. For the exogenous rate method, annual MFP growth is 0.26% and annual capital composition growth is 1.05%. These differences in the unweighted average MFP across industries in the endogenous and the exogenous methods are much larger than reported in Tables 3 and 4 for the total business sector.

Figure 12 shows that there is a large variation in the endogenous rates of return across industries. The endogenous rate of return is higher than the exogenous rate of return in almost all industries, and appears reasonable for most industries. However the endogenous rate of return is extremely large and questionable in the private health care industry (industry 33).

Figure 13 shows that the endogenous rate method yields lower capital composition growth and lower MFP growth in almost all industries. This is a result of the relatively high endogenous rate of return.

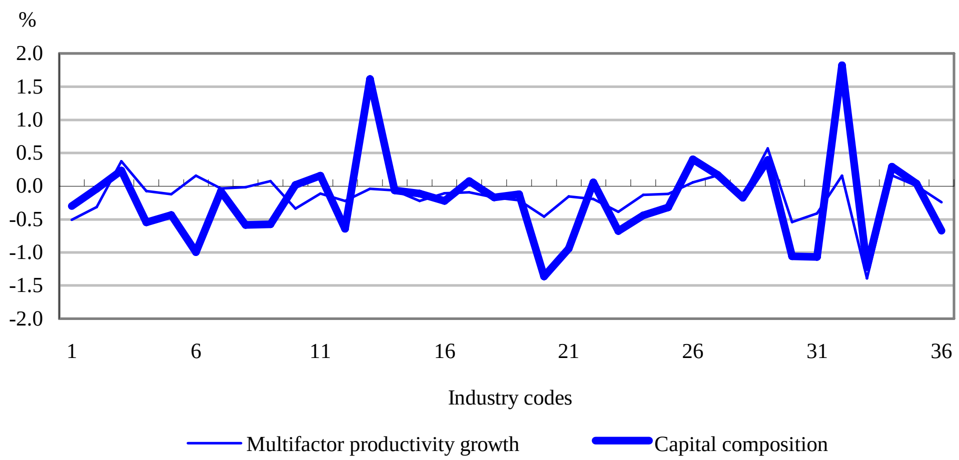
The alternative capital gain specifications also have an important effect on the capital composition and MFP growth estimates at the industry level. Capital composition growth is higher and MFP growth lower for the specifications that account for capital gains (M1, M2, M4 and M5) than for the specifications that do not (M3 and M6). This pattern is pervasive across individual industries, as shown in Figure 14.

Figure 12 Difference between average endogenous and exogenous rates of return over the 1981 to 2001 period by industry



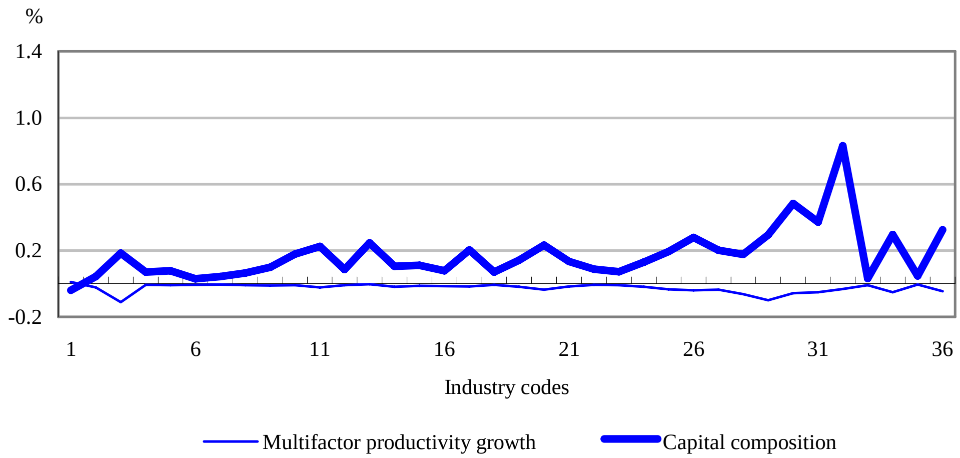
Source: Statistics Canada, Canadian Productivity Accounts.

Figure 13 Difference in annual capital composition and multifactor productivity growth between the endogenous and exogenous method by industry



Note: Annual capital gains are included in the user cost estimation.
Source: Statistics Canada, Canadian Productivity Accounts.

Figure 14 Difference in annual capital composition and multifactor productivity growth between the endogenous rate method with and without capital gains by industry



Source: Statistics Canada, Canadian Productivity Accounts.

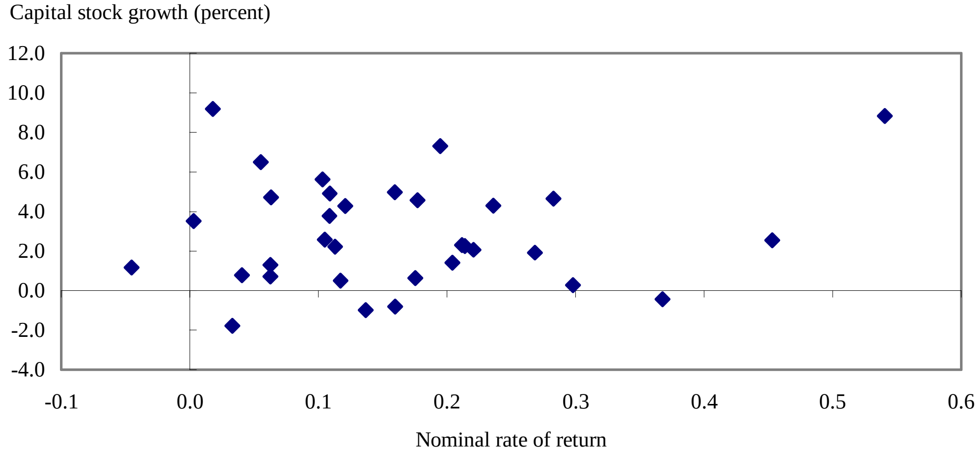
The bottom panel of Table 10 presents a simple average of the standard deviation of the rate of return, the cost share of capital, year-to-year growth in capital composition and MFP over the 36 industries of the Canadian business sector. We find that the exogenous rate of return is much more stable than the endogenous rate of return. The standard deviation of the endogenous rate of return is about 2.2 times larger than the standard deviation of the exogenous rate of return.

The relatively large volatility of the endogenous rate of return leads to the large volatility of the rental price of capital. The standard deviation of the year-to-year growth in the rental price of capital is much larger for the endogenous rate method than for the exogenous rate method.

The relatively large variation of the endogenous rate of return may lead us to expect that the endogenous rate of return specification will yield a more volatile MFP and capital composition growth than the exogenous rate of return specification. The results in Table 10 stand in a sharp contrast to the expectation. We find that there is little difference in volatility of year-to-year MFP growth between the endogenous and exogenous rate specifications. The endogenous rate of return specification yields a more stable capital composition growth than the exogenous rate of return specification. The standard deviation of year-to-year MFP growth obtained from the endogenous rate method is almost identical to the one from the exogenous rate method. The standard deviation of annual growth rate in capital composition is smaller for the endogenous rate of return specification. The largest difference is 42% between M3 and M6. The smallest difference is 31% between M1 and M4.

The alternative capital gains specifications have little effect on the volatility of annual MFP growth but have an effect on the volatility of annual capital composition growth. The assumption of zero real capital gains (M3 and M6) yields a more stable capital composition series than the assumption of real capital gains from asset price changes (M1, M2, M4 and M5), particularly for the endogenous rate specification.

Figure 15 Scatter plot of capital stock growth against endogenous rate of return



Source: Statistics Canada, Canadian Productivity Accounts.

The smoothing of capital gains in the user cost calculation has little effect on the volatility of capital composition and MFP growth at the industry level.

The endogenous rate of return specifications leads to a high incidence of negative estimates of the user cost of capital. The number of negative user costs of capital is largest for the endogenous rate of specifications M1 and M2. For those two specifications, 0.77% of all user costs of capital for 21,600 combinations of 30 assets, 36 industries and 20 years (1981 to 2001) are negative.

In summary, the endogenous method for estimating multifactor productivity growth continues to exhibit the same advantages at the industry level as it did at the aggregate level—with one caveat. Use of the endogenous rate produces some specific anomalies. However, we regard this as an advantage rather than a disadvantage of the endogenous technique. By confronting the industry estimates of surplus with industry estimates of capital stock, both of which come from different sources within the System of National Accounts, this technique allows analysts to improve the consistency of the industry accounts.

5.3.1 Aggregation of capital services over industries

The endogenous rate of return is found to display a large variation across industries. There are two explanations for this cross-industry variation that depend upon alternate views of the world.

First, if markets are felt to operate quickly, capital should earn approximately the same rate of return in all industries. In this case, the observed difference in earned rates of return across industries will reflect random measurement errors in investment, capital stock and capital income. This would suggest that we should use a constant average rate of return to calculate the user cost of capital services at the industry level. The use of such a common rate of return will reduce measurement errors in capital services estimates that would be otherwise imported from the underlying data.

On the other hand, differences across industries in rates of return may be real and systematic. The difference in the rate of return to capital may reflect the barriers to mobility of capital across industries and indicate that adjustments that bring rates of return together do not happen quickly. In this case, we should use the industry-specific rate of return to calculate the user cost of capital services at the industry level.

There is a positive correlation between the average endogenous rate of return and capital stock growth over the 1981 to 2001 period across industries (Figure 15). When we estimate a regression that relates capital stock growth to the average rate of return, we find that the coefficient on the average rate of return is positive and statistically significant at the 5% level.

This suggests that the difference in the rate of return across industries is real and capital tends to move towards those industries that earn relatively high rates of return. In these cases, it has been argued that we should use the industry-specific return to calculate the user cost of capital and that aggregate capital services should then be calculated by aggregating capital service across industries (Jorgenson, Gollop and Fraumeni, 1987). This approach takes into account the difference in the rate of return across industries and avoids the assumption of perfect mobility of capital inputs across industries.

To do so, the aggregate capital input is defined here as a Törnqvist aggregation of capital services across industries

(22) $$\Delta \ln K' = \sum_j \bar{v}_j \Delta \ln K_j ,$$

where $\bar{v}_j$ is the two-period average share of industry *j* in the value of aggregate capital input. The capital services input of industry *j* (K_j) is calculated as an Törnqvist aggregation of all types of capital input:

(23) $$\Delta \ln K_j = \sum_k \bar{v}_{kj} \Delta \ln K_{kj} ,$$

where $\bar{v}_{kj}$ is the two-period average share of asset k in the value of capital services in industry j.

The approach used previously for calculating the aggregate capital services is based on the assumption that capital input earns the same return in all industries. As the rate of return is the same across industries, the aggregate capital services of asset *k* are the sum of capital services across industries:

(24) $$K_k = \sum_j K_{kj}$$

and the aggregate capital services is defined as a Törnqvist index of all types of capital inputs:

(25) $$\Delta \ln K = \sum_k \bar{v}_k \Delta \ln K_k \ .$$

The difference between two alternative estimates of aggregate capital services can be written as:

(26) $$\Delta \ln K' - \Delta \ln K = \sum_k \bar{v}_k \left(\sum_j \bar{w}_{kj} \Delta \ln K_{kj} - \Delta \ln K_k \right),$$

where $\bar{w}_{kj}$ is the two-period average share of capital input k in industry j in the total value of capital input k.

This difference measures the effect of the reallocation of capital services across industries. It is positive when there is a reallocation of capital services towards the industries with relatively higher rates of return and implicitly higher marginal products of capital.

Table 11 Alternative estimates of annual capital services and multifactor productivity growth in the business sector, in percentage

	M2 and industry-specific rate	M2 and same rate of return
Period 1961 to 1981		
Capital services	6.19	5.35
Capital composition	1.60	0.77
Multifactor productivity growth	0.59	0.91
Reallocation of capital	0.83	…
Period 1981 to 2001		
Capital services	3.55	3.09
Capital composition	1.52	1.06
Multifactor productivity growth	0.12	0.31
Reallocation of capital	0.46	…

… not applicable
Source: Statistics Canada, Canadian Productivity Accounts.

Table 11 presents the two alternative estimates of aggregate capital services and capital composition for the total business sector over the periods 1961 to 1981 and 1981 to 2001 using the endogenous rate and 5-year smoothed capital gains. We find that there is a large difference between the two estimates of aggregate capital services. The reallocation of capital services across industries accounts for 52% of capital composition growth over the 1961 to 1981 period and 30% of capital over the 1981 to 2001 period. The MFP growth rate is reduced accordingly.

6. *Conclusion*

The differences in the estimates of multifactor productivity (MFP) that have been provided in this paper suggest that assumptions matter. The alternate methods considered here yield a range of estimates. When faced with differences, the statistician is forced to differentiate among the underlying assumptions discarding those which are less acceptable, either conceptually or practically.

Among the more problematic assumptions considered here is the inclusion of capital gains in the user cost of capital formula. Harper, Berndt and Wood (1989) have noted that the capital gains term is justified on theoretical grounds but that it was probably best to include only a smoothed term in the analysis, arguing that non-smoothed series have so much volatility that it is difficult to believe that firms will attempt to take them into account. Or more importantly, we could argue that the marginal productivity of capital cannot be brought into line with rapid changes in a rental price of capital, that is engendered by capital gains as commonly measured.

Despite concerns with the inclusion of a capital gains term, our findings show, however, that moving from a variant that includes instantaneous expectations to a moving average has little impact on the estimated MFP. What is far more important is the inclusion or exclusion of the capital-gains term itself. Whether we adopt the endogenous or exogenous rate methods, moving from the term that ostensibly contains no capital gains to the term that contains an expression for smoothed capital gains decreases the estimated MFP by 20% to 30%.

Our method of including capital gains in most of the variants investigated in this paper is the conventional one—that involves calculating the rate of changes in the prices of investment goods over time and calculating the rate of change in these prices. And we agree with the literature that theoretically some form of capital gain should be included. But we argue that it would be best to have some guidance from markets themselves rather than have to make arbitrary assumptions as to how to translate these changes into expectations of long-run changes in the price of capital.

Since it is essentially arbitrary whether we include or exclude capital gains, we can place a bound on our estimates by comparing the difference between M2 and M3 or M5 (M5*) and M6. For the period, 1981 to 2001, these differences are 0.10 percentage point or about 30% of the M5 MFP productivity estimate. The difference is 23% for the endogenous rate. For the period 1961 to 1981, the ranges are smaller and are around 4% to 10%.

The other major decision that an analyst needs to make is between endogenous and exogenous rates. Here, the decision is not helped by theoretical considerations. Differences between the two methods may be expected because each approximates the underlying concept in a different way. Multifactor productivity growth is estimated as the difference in the rate of increase in actual output and the rate of increase in output that would have been expected from the increase in factors devoted to production. The endogenous rate method calculates the expected rate of increase in output by calculating the rate of increase across a subset of all assets and then weights this increase by the average marginal revenue product for all assets. The exogenous rate method calculates just the weighted average for the same subset of assets and then weights them by the marginal revenue product of only these assets. The MFP rate calculated from the exogenous rate will be higher than the correct measure to the extent that it does not account for the missing asset that is indirectly taken into account (though perhaps imperfectly) by the endogenous method. And the endogenous rate estimate will differ from the real measure—depending on the difference between the average growth of included and excluded assets.

But of course, the endogenous rate that depends upon assumptions about constant returns to scale and competitive markets may be incorrect if these assumptions do not hold approximately. And those who are worried about the existence of scale economies and market imperfections may feel more comfortable with the exogenous technique.

Instead of deciding the issue based on theoretical considerations, we could ask whether one or other of the alternatives is more attractive when the empirical results are examined. We have seen that the user cost of capital is most volatile for the exogenous rate specification that accounts for annual capital gains. It is only when asset price changes are excluded that the exogenous rate method yields a stable series of the cost of capital. In the same vein, the endogenous method provides a more stable cost share. The stability of the cost share series, arising from the endogenous alternative, is an advantage if a proxy for marginal product is desired that follows a stable time path. Finally, the endogenous rate also is procyclical while the exogenous rate is not. Procyclicality may be an advantage because it allows a multifactor productivity measure to partially take into account the fact that considerable amounts of capital are not being employed in recession.[20]

Nevertheless, the choice between the two techniques may not be important if they yield basically the same results. Consider the differences between the internal and the external rates (M3 and M6). It is worth pointing out that the differences between these two MFP estimates are not large. The endogenous MFP rate is about 14% below the exogenous MFP rate in the period 1961 to 1981 and 12% below in the period 1981 to 2001—within the margin of error that one would expect from misstating the exogenous rate of return by 25%.

Some of this difference may arise because the rate of return embodied in the exogenous rate is lower than in the endogenous rate. And we know from our experiments with alternative estimates for the exogenous rate of return that the lower the rate of return, the larger is this difference. Therefore, any evaluation of the extent to which the exogenous as opposed to the endogenous rate is correct needs to ask whether the exogenous rate of return has been correctly calculated.

For this paper, we have discarded a practice that is often followed of choosing some combination of rates on debt of various maturities—because doing so avoids the issue that equity is such an important component of capital. And when we do so, we find the rate of return so estimated is close, on average, to the endogenous rate—and tracks it closely over time.

While certain improvements in the exogenous rate of return could no doubt be made, there is little point in doing so simply to bring the two MFP rates together. For there are legitimate reasons to believe that there may be differences between the two. As noted earlier, they are different measures.

20. For more discussion of these issues, see Berndt and Fuss (1986), and Hulten (1986).

The quality of any statistical program depends upon the extent to which the summary statistics are fit for use. Meeting this objective requires that agencies give users guidance on appropriate use. Generally, only point estimates of MFP are provided by statistical programs. The effect of using perfectly good alternate assumptions for the production of summary statistics is rarely investigated. This paper addresses this gap for MFP estimates. Doing so is important when users employ MFP measures in cross-country comparisons. Users need to be aware that the MFP estimates are point estimates with a confidence interval around them. That confidence interval cannot easily be calculated using classical statistical techniques. But we can give users an idea of how our estimates would change if we employed alternate assumptions.

We have shown that the point estimates that we produce here could very well vary by 20%, either because of the choice of the endogenous over the exogenous technique or because of the way in which capital gains are measured. When users employ MFP estimates for cross-country studies, these types of confidence intervals should be kept in mind.

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