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Unmeasured Investment and the Puzzling U.S. Boom in the 1990s*

Ellen R. McGrattan
Federal Reserve Bank of Minneapolis
and University of Minnesota

Edward C. Prescott
Arizona State University
and Federal Reserve Bank of Minneapolis

ABSTRACT

The basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated. For the 1990s, the model predicts a depressed economy, when in fact the U.S. economy boomed. We extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that the 1990s are not puzzling in light of this new theory. There is compelling micro and macro evidence for our extension, and the predictions of the theory are in conformity with U.S. national products, incomes, and capital gains. We use the theory to compare current accounting measures for labor productivity and investment with the corresponding measures for the model economy with intangible investment. Our findings show that standard accounting measures greatly understate the boom in productivity and investment.

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The basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity (TFP), and taxes are incorporated.[1] The behavior of the 1990s, however, is strikingly at variance with this model, particularly in comparison with the boom in hours, but also in comparison with the behavior of most aggregate series that business cycle theorists study. To put it succinctly, the model predicts a depressed 1990s economy, when in fact it boomed.

In this paper, we extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that, in light of this new theory, the 1990s are not puzzling. Intangible investment is excluded from GDP because it is difficult to measure. Examples include research and development (R&D), advertising, and investments in building organizations. Some intangible investment is financed by owners of capital and is *expensed* rather than capitalized. Some is investment in *sweat equity* financed by worker-owners who allocate effort and time to their business and receive compensation at less than their market rate. These investments are made with the expectation of realizing future profits or capital gains when the business goes public or is sold.

We have found compelling evidence that both of these types of unmeasured investment were abnormally high in the 1990s. The National Science Foundation's report of R&D investment shows that R&D relative to GDP grew by 30 percent between 1994 and 2000.[2] The Current Population Surveys of the U.S. Department of Labor show a shift of labor into information technology–related and managerial occupations, with greater opportunities for business owners to make capital gains on expensed and sweat investment. If we look at patterns of national incomes, we also find evidence of abnormally high investment at the macroeconomic level Corporate profits were falling as output was rising, suggesting that investment in R&D and other intangible capital was abnormally high. Similarly,

[1] See, for example, Chen, İmrohoroğlu, and İmrohoroğlu (2007) and our technical appendix (McGrattan and Prescott 2007).

[2] Corrado, Hulten, and Sichel (2005, 2006) include other categories of expenditures and conclude that over the 1990s, total intangible investment was large and was increasing as a share of business output.

compensation per hour was low during the 1990s, suggesting that sweat investment was likewise abnormally high.

The fact that *measured* factor incomes were low when output and hours were booming is consistent with a theory that differentiates economic income and measured income, which need not move together and indeed did not move together in the 1990s. To uncover what actually happened during the 1990s, we use our extended theory and U.S. national income and product account (NIPA) data. Specifically, we incorporate intangible investment into an otherwise basic neoclassical growth model. Two technologies are available in the business sector: one for producing final goods and services and one for producing intangible capital. We use the extended model to determine the path for intangible investment, and show why including this type of investment is critical for understanding the boom in the U.S. economy in the 1990s.

We allow the rates of technological change to differ across these two technologies, thus allowing for a technology boom in the sector producing intangible capital—a boom like the one that occurred in the United States. During the 1990s, rapid technological advancements were being made in industries that are relatively intensive in producing intangible capital, such as those related to information technology. Two notable pieces of evidence (Doms 2004) are what happened to Intel processor speeds, which increased 4.6 percent per month in the period 1997–2000, and fiber-optic throughput, which rose from 2.5 gigabits to 400 gigabits per second between 1995 and 2000. Given that households equate wages and rental rates across production activities, we have a way to identify the TFP paths and to estimate the magnitude of intangible investment. We estimate that net intangible investment in the business sector was about 3 percent of GDP prior to 1990, rose to over 8 percent of GDP in the 1990s, and then returned to the level of the early 1990s in 2001.

We could have modified the basic model by introducing large and variable shocks to preferences for leisure, which is a common practice in business cycle research.[3] However,

[3] See, for example, Hall (1997), Chang and Schorfheide (2003), Galí (2005), Comin and Gertler (2006), Galí, Gertler, and López-Salido (2007), Ireland and Schuh (2007), Kahn and Rich (2007), and Smets and Wouters (2007) who point out that these shocks proxy for variations in tax rates and other labor

doing so would have violated two criteria that we require to successfully resolve the puzzling 1990s boom—or any other puzzle, for that matter. The first is the *input justification criterion*. By this, we mean that we require our exogenous inputs to be consistent with micro and macro empirical evidence. A large rise in factor productivity in the sector producing intangible capital is consistent with the U.S. technology boom and the shifts in employment into high-technology occupations and managerial positions. A large rise in preferences for leisure, on the other hand, cannot be justified by any observations on tax rates, tax credits, or welfare benefits.

The second of our two criteria for successfully resolving of the puzzling 1990s boom is the *prediction criterion*. At a minimum, to satisfy this criterion, a theory's predictions must not be counterfactual. A stronger requirement—one that is satisfied by our theory—is to make correct predictions for data that were not used to set parameters or exogenous inputs. Thus, we do not follow the widely-used practice in the business cycle literature of including the same number of exogenous inputs as observed time series, which is done to ensure a perfect fit between data and theory.

Here, there are two sequences of TFP parameters that are free in the analysis and many time series that must be in conformity with the theory. We find that the equilibrium paths of our extended theory are in close conformity with time series of both NIPA products and incomes and, most importantly, with the increase in capital gains that occurred in the second half of the 1990s. This increase was large, with the average real gains going from 6 percent of GDP in the period 1953–1994 to 12 percent of GDP in the period 1995–2003. Data on factor incomes and capital gains are not used to identify the TFP parameters. In contrast, a theory based on a large shift in preferences for leisure during the 1990s does not account for the observed changes in factor incomes and capital gains.[4]

After demonstrating that the model's predictions are in conformity with U.S. time series, we use the model to compare current accounting measures for investment and labor

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market distortions, which are especially important in accounting for changes in hours. In McGrattan and Prescott (2007), we show that labor tax rates do account for much of the cyclical variation in hours prior to the 1990s but not in the 1990s.

[4] See McGrattan and Prescott (2007) for details.

productivity with corresponding measures that include expensed and sweat investment. Solow's (1987) remark that "you can see the computer age everywhere but in the productivity statistics" is pertinent for our findings. If expensed and sweat investments are included, the model predicts an earlier and larger boom in productivity than current accounting measures would indicate. Based on this prediction, we conclude that ignoring these two types of intangible investment distorts the true picture of the U.S. economy in the 1990s.

Our findings show that standard productivity measures greatly understate the actual rise in labor productivity whether we consider the overall economy or the business sector. Our analysis is not subject to the criticism of Brynjolfsson and Hitt (2000) who point out that intangible investment is "not well captured by traditional macroeconomic measurement approaches." Here, we explicitly model the intangible investment. Accounting for intangible investment, we find that the boom in business productivity began earlier and was bigger than standard statistics show. Over the period 1993–2000, the difference in labor productivity growth due to the inclusion of intangible investment is 0.8 percent per year.

The paper is organized as follows. In Section 1, we establish that basic theory—abstracting from unmeasured investment—generates strongly counterfactual predictions. In particular, the basic growth model does not generate a boom in the 1990s. If we extend the model to have two sectors, a private business sector and a non-business sector, then there is some improvement in the model's predictions given that business TFP boomed in the late 1990s. However, the rise in TFP is too small and occurs too late to account for the boom in hours that began in 1992. In Section 2, we summarize the evidence of increased intangible investment which motivates our extension of the basic theory. The extended theory includes expensed and sweat investment, and in Section 3, we assess its predictions. In Section 4, we reevaluate the performance of the U.S. economy in the 1990s through the lens of the extended theory. Conclusions are found in Section 5.

1. Predictions of the Basic Theory without Intangible Investment

Our starting point is the basic growth model used in the study of business cycles. This model abstracts from intangible investment. We treat TFP, tax rates, and population exogenously. We then use U.S. data for the 1990s to establish that there are large deviations from the basic model, indicating that this model must be abstracting from something important. We show that the nature of the deviations points to unmeasured investments.

1.1. The Basic Growth Model

In the standard one-sector growth model, given initial capital stock k_0, the problem for the stand-in household is to choose consumption c, investment x, and hours h to maximize

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t \tag{1.1}$$

subject to the constraints

$$\begin{aligned} c_t + x_t = r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t) \\ - \tau_{dt}\{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t)\} \end{aligned} \tag{1.2}$$

$$k_{t+1} = [(1-\delta)k_t + x_t]/(1+\eta), \tag{1.3}$$

where variables are written in per capita terms and $N_t = N_0(1+\eta)^t$ is the population in t. Capital is paid rent r_t, and labor is paid wage w_t. Households discount future utility at rate β, and capital depreciates at rate δ. Taxes are levied on consumption at rate τ_c, labor income at rate τ_h, tangible capital (that is, property) at rate τ_k, profits at rate τ_p, and capital distributions at rate τ_d. Note that taxable income for the tax on profits is net of depreciation and property tax, and taxable income for the tax on distributions is net of property tax and profits tax.

The aggregate production function is

$$Y_t = A_t F(K_t, H_t), \tag{1.4}$$

where capital letters denote aggregates. The parameter A_t is TFP that varies over time. The firm rents capital and labor. If profits are maximized, then the rental rates are equal

to the marginal products. The goods market clears so $N_t(c_t + x_t) = Y_t$. Here, c includes both private and public consumption, and x includes both private and public investment.

We now show that this basic growth model generates grossly counterfactual predictions during the 1990s. To do so, we compute the model's equilibrium path with households having perfect foresight of future changes in tax rates, TFP, and populations. In Appendix A, we discuss our U.S. data sources and the adjustments we make to construct the empirical counterparts of the model variables. In Appendix B, we describe and motivate the parameterization we use for the model. The parameter values used to compute the equilibrium path here are summarized in Table B.1 under "One-Sector Model, No Intangible Investment." The paths for TFP and tax rates are reported in Table B.2.[5]

Our estimate for TFP is U.S. GDP divided by $F(K, H)$, with K equal to the total stock of U.S. tangible capital and H equal to total hours. Here, $F(K, H) = K^{\theta} H^{1-\theta}$. The tax rate changes we consider are variations in the labor tax rates τ_{ht} and consumption tax rates τ_{ct}, as constructed in Prescott (2004) with data from U.S. national accounts. During the 1990s, there was little change in legislation affecting capital taxation, and therefore we simply fix the rates τ_{kt}, τ_{pt}, and τ_{dt}.

The utility flow function is

$$U(c, h) = \log(c) + \psi \log(1 - h),$$

which is standard in the business cycle literature. We choose the level of capital tax rates, the depreciation rate δ, and the utility parameter ψ so that the model's consumption share, investment share, factor inputs, and tax revenues are consistent with U.S. levels in 1990. (See appendices A and B for details.)

In Figure 1, we plot the model's predicted per capita hours of work along with the U.S. actual per capita hours, indexed so that 1990 equals 100. The difference between the series is striking. Actual per capita hours rose 8 percent between 1992 and 1999, whereas the predicted series falls significantly during the same period.

[5] In McGrattan and Prescott (2007), we demonstrate that the perfect foresight assumption is innocuous by comparing the results to stochastic simulations. We also demonstrate the robustness of the results by varying parameters.

In Figure 2, we plot the model's predicted output along with U.S. real GDP. Both series are adjusted for population and a secular trend of 1.02^t. Although the boom in output was not quite as large as the boom in hours, the model predicts that the economy should have been depressed. This counterfactual prediction arises from the fact that the tax rates on labor rose during the 1990s and economy-wide TFP was below trend during most of the decade.

The basic model has neutral TFP change with respect to the business and non-business sectors. In fact, TFP change was non-neutral for these sectors. A question that arises is whether modeling this non-neutrality of TFP change could significantly narrow the large deviation from theory. In the two-sector version of the model, households solve the same problem, although they now have to allocate capital and labor to two technologies, "business" and "non-business." (See McGrattan and Prescott (2007) for model details.)

In Appendix A we describe how we categorize business and non-business activity. In Appendix B, we describe and motivate the parameterization for this version of the model. Parameter values used to compute the equilibrium path here are summarized in Table B.1 under "Two-Sector Model, No Intangible Investment." As before, the paths for TFP and tax rates are exogenous. These are reported in Table B.2. We assume that households in the model make exactly the same choices for non-business activity as U.S. households. We simply set the values of non-business hours, investment, and value added exogenously to U.S. levels. We treat this sector exogenously because it is quite small compared to the business sector. Furthermore, if there are any deviations between the model's predictions and U.S. data, we can attribute these deviations to our model of the business sector, which did have a boom in TFP.

We find that modeling the non-neutrality of TFP in business and non-business activity does not resolve the puzzling 1990s boom. Model predictions are still far from observations. In particular, output is still predicted to be below trend throughout the decade, and per capita hours are still predicted to be low. (Figures of these series and others are in McGrattan and Prescott 2007.) The boom in business-sector TFP is too small and too late. Clearly, something else gave rise to the puzzling behavior of the U.S. economy in the

1990s.

1.2. Investigating the Deviation

Why are the model's predictions so far off for the basic neoclassical growth model? The main reason is the behavior of TFP and tax rates. Given the behavior of these inputs, the model predicts an after-tax real wage, $(1-\tau_h)w$, below its secular trend. Not surprisingly, then, the model predicts that hours are low and output is below trend.

We turn next to evidence that using the wrong measure of output and understating labor productivity and, therefore, the after-tax real wage accounts for the large deviation between theory and data. The mismeasurement stems from abnormally large unmeasured intangible investment during this period. Standard measures of output growth are distorted when the importance of intangible investments grows.

2. Evidence of Increased Intangible Investment

We present two types of evidence that unmeasured intangible investment was abnormally large during the 1990s. One type of evidence is related to the behavior of NIPA compensation and profits, the other type to the technology boom going on during the period. Because intangible investments are expensed in the NIPA, measurements of factor incomes are understated to a greater extent in periods when these investments are high. We show that that was true for the 1990s. We then present evidence that, in fact, during the technology boom, the level of investment was indeed high and led to large capital gains that are missed by the NIPA's income measure.

2.1. Low Compensation and Corporate Profits

If all incomes were included in the national accounts, we would expect to see compensation per hour and profits to be high during a boom. But an examination of the U.S. national accounts reveals that compensation per hour and profits were low during the boom period, suggesting that unmeasured expensed and sweat investment was abnormally high.

In Figure 3, we plot average weekly hours of work for the noninstitutional population, age 16 to 64, which is the same series plotted in Figures 1 and 2. We also plot the wage rate corresponding to these hours, which is computed as follows. We take NIPA compensation and deflate it by the GDP deflator. We then correct for population growth by dividing real compensation by this population. Finally, because there is technological growth, we divide the wage rate by the factor 1.02^t, where t indexes time.[6] For all of the 1990s, NIPA real compensation per hour detrended in this way is below the 1990 level, despite the boom in hours.[7]

In Figure 4, we compare NIPA GDP and corporate profits, both deflated by the GDP deflator and adjusted for population and a secular trend of 1.02^t. We see that profits fall in the late 1990s when GDP, R&D, and capital gains are high.

2.2. The Technology Boom

We have found additional direct evidence that unmeasured investment was abnormally high in the 1990s. The 1990s was a period of rapid technological advances. Companies were increasing R&D, and the payoffs were evident in increased initial public offerings (IPOs) and mergers and acquisitions. Further, the rise in hours was particularly large for the more educated in occupations in which people make large sweat investments on average.

One indicator of increased intangible investment is increased funding of R&D, which is expensed by corporations. The National Science Foundation (NSF) (1953–2003) reports that industry R&D increased 68 percent between 1994 and 2000, whereas GDP rose only 39 percent. A significant fraction of the company-funded R&D was done by companies in information technology industries. Using data from the NSF, Doms (2004) estimates that the information technology share of company-funded R&D averaged 27 percent in the period 1997–2001.

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[6] The particular choice of 1.02 for the secular trend does not affect any results, but makes it easier to see the patterns in this and later figures.

[7] In earlier work (McGrattan and Prescott 2005b), we abstract from sweat equity investment and treat NIPA compensation as true labor income. Doing so reduces the estimate of intangible investment.

Another indicator of abnormally large intangible investment is the dramatic increase in IPOs and mergers and acquisitions. According to data from the Thomson Financial Securities Data Corporation (SDC) database (also available in Ritter and Welch 2002, Table 1), gross proceeds from IPOs were significantly higher in the 1990s than in the 1980s. Gross proceeds of IPOs averaged $8.2 billion over the period 1980–89 and $30.9 billion over the period 1990–99. Large increases in the value of existing equity, and therefore large capital gains, are typically associated with IPOs. Because these gains are not included in NIPA, NIPA incomes understate true income. Other related evidence available from the SDC database is the volume of announced mergers and acquisitions. The volume rose from $0.6 trillion in 1994 to $3.4 trillion by 2000. As in the case of IPOs, the accrued capital gains are not included in NIPA measures of income.

We have presented evidence that business owners made abnormally large unmeasured investments and accrued abnormally large capital gains during the 1990s boom. We now present evidence that the hours of certain categories of workers that tend to have a disproportionate number of sweat investors increased disproportionately in the 1992–2000 period. These categories include those who have had at least a year of college education and who work as managers, proprietors, computer analysts, and in certain financial occupations. We use Current Population Survey (CPS) data to determine hours worked in these categories.[8] These categories of workers accounted for 50 percent of the increase in hours between 1992 and 2000, even though they accounted for only 10 percent of hours in 1992.

The fall in hours in 2000 was coincident with a fall in information technology investment, which occurred partly because of regulatory factors. According to Couper, Hejkal, and Wolman (2003) and Doms (2004), the telecommunications sector contributed significantly to the *technology bust*. Part of the problem was that the demand for long-haul fiber capacity was not as great as anticipated. Another problem was what Federal Communications Commission chairman Michael Powell called "the problem of legal instability in the

[8] Here, we are referring to data compiled from the March supplement of the CPS survey. We split workers into two groups: those with variable EDUCREC greater than or equal to 8 and variable OCC in the set {4, 7, 9, 13, 14, 15, 18, 21, 22, 34, 37, 64, 65, 229, 23, 24, 25, 225} and the remainder. See Ruggles et al. (2004) for more details.

court system" (Powell 2002). The Telecommunications Act of 1996 brought competition to local telephone service, but it generated many legal battles as well. According to Couper, Hejkal, and Wolman (2003), the regulatory uncertainty has discouraged telecommunications companies from undertaking the large-scale investments needed for fiber-optic service to individual residences.

In summary, we find compelling evidence that intangible investment was abnormally high during the 1990s. This motivates an extension of the basic model with both tangible and intangible investment and technological change that gives rise to an increase in the importance of intangible investment.

3. Predictions of the Extended Theory with Intangible Investment

We now extend the basic growth model to include intangible investment and non-neutral technological change with respect to production of final goods and services and production of new intangible investment goods. Intangible investments are made by businesses, so the extended model distinguishes business and non-business activity. We start by describing the technologies available to businesses, the optimal business size, and the aggregate production technology. The household problem remains the same as in Section 2 except for an additional investment choice. We examine the extended model's predictions and show that these predictions are in conformity with U.S. observations during the 1990s.

3.1. Extensions

The aggregate production technology is characterized by the two aggregate production relations:

$$y_{bt} = A_t^1 (k_{Tt}^1)^{\theta_1} (k_{It})^{\phi_1} (h_t^1)^{1-\theta_1-\phi_1} \tag{3.1}$$

$$x_{It} = A_t^2 (k_{Tt}^2)^{\theta_2} (k_{It})^{\phi_2} (h_t^2)^{1-\theta_2-\phi_2}. \tag{3.2}$$

Firms produce business output y_b using their intangible capital k_I, tangible capital k_T^1, and labor h^1. Firms produce intangible capital x_I—such as new brands, new products R&D, patents, etc.—using intangible capital k_I, tangible capital k_T^2, and labor h^2.

Note that k_I is an input to both business sectors; it is not split between them as is the case for tangible capital and labor. A brand name is used both to sell final goods and services and to develop new brands. Patents are used by the producers and the researchers. The aggregation theory underlying this technology is developed in Appendix C.

Given (k_{T0}, k_{I0}), the stand-in household maximizes

$$E\sum_{t=0}^{\infty}\beta^t[\log c_t + \psi\log(1-h_t)]N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} = {} & r_{Tt}k_{Tt} + r_{It}k_{It} + w_t h_t + \zeta_t \\
& - \tau_{ct}c_t - \tau_{ht}(w_t h_t - (1-\chi)q_t x_{It}) - \tau_{kt}k_{Tt} \\
& - \tau_{pt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt}k_{Tt}\} \\
& - \tau_{dt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - x_{Tt} - \chi q_t x_{It} - \tau_{kt}k_{Tt} \\
& \quad - \tau_{pt}(r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt}k_{Tt})\}
\end{aligned}$$

$$k_{T,t+1} = [(1-\delta_T)k_{Tt} + x_{Tt}]/(1+\eta) \quad (3.3)$$

$$k_{I,t+1} = [(1-\delta_I)k_{It} + x_{It}]/(1+\eta). \quad (3.4)$$

As before, all variables are in per capita units and there is growth in population at rate η. Consumption c includes both private and public consumption, and tangible investment x_T includes both private and public investment. The relative price of intangible investment and consumption is q.[9] The rental rates for business tangible and intangible capital are denoted by r_T and r_I, respectively, and the wage rate for labor is denoted by w. Inputs are paid their marginal products. The tax system is the same as in the standard model. Other income is denoted by ζ and is exogenous in the household's decision problem. Other income includes government transfers and non-business capital income net of taxes and investment. Non-business labor income is included in wh.

As before, we treat hours, investment, and output in the non-business sector exogenously because this sector is not important for the issues being addressed. To be precise,

[9] For some purposes, modeling the changes in technology that induce changes in the relative price of c_t and x_{Tt} is important. See, for example, Hornstein and Krusell (1996) and Greenwood, Hercowitz, and Krusell (1997). For our purposes, it is reasonable to abstract from this change in technology.

we set the levels of non-business hours $\bar{h}_n$, investment $\bar{x}_n$, and output $\bar{y}_n$ in the model's non-business sector equal to U.S. levels. Measured output, which corresponds to GDP, is the sum of y_b and $\bar{y}_n$. Measured tangible investment is the sum of business tangible investment x_T and non-business tangible investment $\bar{x}_n$. Measured hours h is the sum of business hours $h^1 + h^2$ and non-business hours $\bar{h}_n$.

Let χ denote the fraction of intangible investment financed by capital owners. The amount $\chi q x_I$ is *expensed investment*, financed by the capital owners who have lower accounting profits the greater this type of investment. The amount $(1-\chi)qx_I$ is *sweat investment*, financed by workers who have lower compensation the greater this type of investment.[10]

Gross domestic product in the economy is the sum of total consumption (public plus private) and tangible investment (public plus private) for business and non-business; in per capita terms GDP is $c + x_T + \bar{x}_n$. Gross domestic income (GDI) is the sum of all labor income less sweat investment $wh - (1-\chi)qx_I$, business capital income less expensed investment, $r_T k_T + r_I k_I - \chi q x_I$, and non-business capital income (which is found residually as the difference between GDP and the other components of GDI).

3.2. A Resolution of Seemingly Low Wages

We showed earlier that there is a large deviation between predictions of the basic one-sector and two-sector growth models and U.S. data. The models predict that after-tax real wages in the 1990s should have been below trend, leading to low per-capita hours and output below its secular trend. With our extended model, the measure of the real wage is different and is consistent with the behavior of output and hours.

The standard model measure of the business real wage is

$$\hat{w}_t = (1-\theta)y_{bt}/(h_t^1 + h_t^2), \tag{3.5}$$

where θ is the capital share, y_{bt} is measured business value added, and $h_t^1 + h_t^2$ is total

[10] In the absence of informational or financial constraints, the choice depends in a knife-edge way on the tax treatment of the expensed and sweat investment. This is analogous to the result of Miller (1977) for the debt-equity choice.

business hours. The problem with the measure of labor productivity on the right side of equation (3.5) is that some hours are used to accumulate intangible capital. The hours used to produce y_{bt} are h_t^1 and, therefore, the real wage measure is

$$w_t = (1-\theta) y_{bt}/h_t^1, \tag{3.6}$$

where $\theta = \theta_1 + \phi_1$ and y_{bt}/h_t^1 is labor productivity in production of final goods and services. The labor input h_t^2 is used to produce output $q_t x_{It}$ and is not part of the labor input in producing y_{bt}. If the relative size of h_t^2 relative to h_t increases, then $w_t/\hat{w}_t$ increases and the percentage understatement of true wages becomes more severe.

The evidence presented earlier suggests that advances in technology were particularly large in activities related to intangible production. This would imply an increase in A_t^2/A_t^1. Our hypothesis is that A_t^2/A_t^1 did indeed increase significantly and that such an increase leads to an increase in the relative hours allocated to producing intangible investments, namely h_t^2/h_t.

3.3. Identifying Total Factor Productivities

In order to identify total factor productivities, the magnitude of the inputs and the outputs to the production functions must be determined. This requires determining the split of hours and tangible capital between two production activities in the business sector and the magnitude of intangible investment and capital.

To determine how much labor is allocated to the two production activities, we use the fact that the after-tax real wage rate is equal to the marginal rate of substitution between leisure and consumption, $\psi(1+\tau_{ct})c_t/(1-h_t)$. We have observations on consumption c, total hours h, business value added y_b, and tax rates. We use these observations to determine hours in production of final goods and services as follows:

$$h_t^1 = \left(\frac{1-\theta_1-\phi_1}{\psi}\right)\left(\frac{1-\tau_{ht}}{1+\tau_{ct}}\right)\left(\frac{y_{bt}}{c_t}\right)(1-h_t). \tag{3.7}$$

Hours in the accumulation of intangible capital is determined residually, $h_t^2 = h_t - h_t^1 - \bar{h}_{nt}$. Equation (3.7) is simply a rewriting of the household's intratemporal condition relating

the marginal rate of substitution between leisure and consumption to $(1-\tau_h)w$ using w in (3.6).

Equating the marginal products of labor in the two activities yields an equation that can be solved for $q_t x_{It}$ as a function of the hours h^1 and h^2 determined above and business value added y_b which is observed,

$$q_t x_{It} = \left(\frac{1-\theta_1-\phi_1}{1-\theta_2-\phi_2}\right)\frac{y_{bt}}{h_t^1}\, h_t^2. \tag{3.8}$$

The allocation of tangible capital across the two activities in the business sector is determined in a similar way. The initial stock $k_{T,1990}$ and the sequence of business tangible investments imply the sequence of stocks $\{k_{Tt}\}$ from (3.3). Equating the marginal products of tangible capital across the two business activities implies

$$k_{Tt}^1 = \left(\frac{\theta_1 y_{bt}}{\theta_1 y_{bt} + \theta_2 q_t x_{It}}\right) k_{Tt},$$

where $k_{Tt}^2 = k_{Tt} - k_{Tt}^1$ is residually determined.

If we have a sequence for the price q_t of intangible investment, we could use the already computed sequence of outputs $q_t x_{It}$ to determine a sequence for intangible investment x_{It}, and, with an initial condition for the stock $k_{I,1990}$, we could use (3.4) to determine the sequence of stocks.[11] But we do not have q_t and, therefore, we use the intertemporal condition

$$1 = \beta\left(\frac{1+\tau_{ct}}{1+\tau_{c,t+1}}\right)\left(\frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)}\right) R_{t,t+1}, \tag{3.9}$$

where $R_{t,t+1}$ is the after-tax return realized by the household investing in intangible capital,

$$\begin{aligned} R_{t,t+1} = \Big\{ & q_{t+1}(1-\chi)(1-\tau_{h,t+1})(1-\delta_I) + (1-\tau_{p,t+1})(1-\tau_{d,t+1})[q_{t+1}\chi(1-\delta_I) \\ & + (\phi_1 y_{b,t+1} + \phi_2 q_{t+1} x_{I,t+1})/k_{I,t+1}]\Big\}/ \\ & \Big\{q_t[(1-\chi)(1-\tau_{ht}) + \chi(1-\tau_{pt})(1-\tau_{dt})]\Big\}, \end{aligned} \tag{3.10}$$

along with the capital accumulation equation and the already identified sequence for $q_t x_{It}$. This relation implicitly identifies q_{t+1} as a function of current variables and q_t. As a

[11] We use the steady-state stock to initialize intangible capital.

terminal condition, we assume that the price in the year following the end of our sample is equal to the price in the last year of our sample.

The final step in identifying the TFP parameters is to use the production functions (3.1) and (3.2) to determine the $\{A_t^1, A_t^2\}$. The resulting sequences for these two TFPs are plotted in Figure 5 along with a more standard measure of business-sector TFP that abstracts from intangible capital: business value added divided by $k_{Tt}^{0.33}(h_t - \bar{h}_{nt})^{0.67}$, where k_{Tt} is tangible capital in the business sector and $h_t - \bar{h}_{nt}$ is total business hours. The latter measure is labeled "U.S. Business Sector." All series are real and divided by 1.02^t.

The standard measure of business-sector TFP shows some acceleration beginning in 1996. The implied TFPs for the model with intangible investment show larger increases that begin earlier. For example, in the sector producing final goods and services, predicted growth in TFP between 1993 and 2000 is 0.7 percent greater per year than is found by constructing the standard measure. In the sector producing intangible capital, the implied growth in TFP between 1993 and 2000 is 2.7 percent greater per year than that found with the standard measure. All three measures show some decline after 2000, which could well have been due to regulatory and legal factors impinging negatively on efficiency.

The patterns of the TFP sequences are consistent with the micro and macro evidence we cite in Section 2. For this reason, we view this theory as one that satisfies our exogenous input justification criterion. We next show that the theory satisfies our prediction criterion.

3.4. Model Predictions

Treating the TFP sequences as exogenous inputs, we compute the equilibrium path of all of the variables and compare them to U.S. data. All of the parameters used in computing the equilibrium are described and motivated in Appendix B and summarized in Table B.1, under the heading "Extended Model, with Intangible Investment," and in Table B.2.[12]

In Figure 6, we display the implied intangible investment as a share of total output,

[12] Income shares listed in Table B.1 are the same across activities. We do sensitivity checks on these shares and other parameters of the model to ensure that the main quantitative results are robust. See McGrattan and Prescott (2007).

by which we mean GDP plus intangible investment. This figure displays the bottom line of our study: the value of this investment is large and increased dramatically in the 1990s. That is precisely what we see in Figure 6. Our estimate of *net* intangible investment—expensed plus sweat—in the business sector is a little over 3 percent of total output—GDP plus intangible investment—in 1990 and rises to nearly 8 percent of total output before returning to the level of the early 1990s.[13]

We now assess the conformity of two sets of predictions. The first are predictions of variables used to identify the sequences of TFP in Figure 5. These series are hours and components of GDP. The second are predictions of variables that are not used to identify the sequences of TFP. These series are NIPA factor incomes and capital gains reported in the U.S. Flow of Funds (1945–2005); the latter is especially important given the central role that capital gains play in our extended theory. We find that the model is in conformity with both sets of predictions.

Internal Conformity

We start with a comparison to total hours and to components of GDP. Note that although we used equilibrium conditions to identify the TFP parameters, the predicted and actual series may differ because we used *only one of the two* intertemporal conditions when determining the TFP paths. Condition (3.9) relates the marginal rate of substitution in consumption between period t and $t+1$ to the after-tax return on investing in intangible capital. The second intertemporal condition, which was not used, relates the marginal rate of substitution in consumption to the after-tax return on investing in tangible capital. If the latter condition is not satisfied by the data, the predicted and actual paths will differ.

Figure 7 shows the results for per capita total hours worked. Unlike the comparable figure with the standard model's predictions (Figure 1), here, the predictions and the actual series track each other closely. The extended model predicts a fall in hours used to produce final goods and services during the 1990s. However, because hours spent building intangible capital rise significantly, the model predicts the large overall increase in hours

[13] The estimate of net intangible investment here exceeds earlier estimates in McGrattan and Prescott (2005a). In earlier work we did not include sweat investment or noncorporate business activity.

worked h.

Similarly, the model and data paths for GDP are close. We plot these paths in Figure 8. The model's prediction for business value added is also close because, in the model, GDP is the sum of business value added y_b and non-business value added $\bar{y}_n$, where $\bar{y}_n$ is preset to be the same as in the United States. In the appendix, we display time series for consumption, tangible investment, and business labor productivity, which are all close in comparison to their U.S. analogues.

External Conformity

Now we consider a more demanding test of the theory: comparing model predictions to observations not used to determine the TFP paths. In particular, we compare predictions for business wage compensation as measured in the NIPA and for business capital gains as measured in the Flow of Funds accounts. We find that the model predicts these series remarkably well.

To compare the model's prediction for NIPA wage compensation in the business sector, we need to construct wages as a national accountant would. Such an accountant, placed in the model economy, would report wage compensation in the business sector as $w_t(h_t^1 + h_t^2) - (1-\chi)q_t x_{It}$, in effect, not including the value of sweat equity investment. In Figure 9, we plot this predicted series along with the actual U.S. series. Both are real series, detrended by 2 percent annually and set equal to 100 in 1990. The two are close. Relative to the 1990 trend level, both the model prediction and the actual wages are up nearly 8 percent in 2000. We note that our choice of $\chi = 0.5$ is relevant for this prediction. The value of χ determines the level of taxation on expensed versus sweat equity, which affects the equilibrium measured compensation. Higher values of χ increase the predicted value of compensation. We selected $\chi = 1/2$ given that we do not have independent evidence of the financing of expensed and sweat equity. In McGrattan and Prescott (2007), we show that our results are not sensitive to the choice of χ unless χ is far from 1/2.

Next, we compare the model's predictions for estimates of the increase in capital gains from expensed and sweat equity to U.S. household holding gains reported in the Flow of

Funds accounts. Those gains are the change in the value of assets outstanding (taken from Table L.100) less the net purchases during the period (taken from Table F.100). If Flow of Funds accountants recorded holding gains for our model households, they would compute differences in the total value of businesses (for which the household is the residual claimant). The value of all businesses in t, V_t, is composed of two parts:

$$V_t = (1 - \tau_{dt})K_{T,t+1} + [\chi(1 - \tau_{dt})(1 - \tau_{pt}) + (1 - \chi)(1 - \tau_{ht})]q_t K_{I,t+1}, \tag{3.11}$$

where capital letters denote aggregates. On the right side of (3.11), the first term is the value of tangible capital and the second is the value of intangible capital. Notice that the price of intangible capital depends on χ, since income to capital and income to labor are taxed differently.

The change in the value V_t of businesses does not exactly reflect the additional income in the model economy. The additional income is $q_t X_{It}$ (in units of the final goods and services). However, during periods with large investments of intangible capital, the increase in holding gains, as defined in the Flow of Funds accounts, is a good approximation to the increase in intangible investment.

So that our estimates are comparable with the U.S. Flow of Funds, we make an adjustment to our model estimate to add foreign gains because our model includes only domestic sectors. Since many domestic corporations have foreign subsidiaries, the value of U.S. corporations includes equity from foreign capital, and the holding gains include gains from this foreign capital. We estimate these gains by assuming that the ratio of after-tax foreign corporate profits (excluding gains) to after-tax domestic corporate profits (excluding gains) is equal to the ratio of foreign to domestic holding gains. With this assumption, our estimate of foreign gains relative to total gains is approximately 23 percent on average for the period 1990–2003.

A significant break in U.S. real holding gains (relative to GDP) occurred in 1995. Before that year, the series averages around 6 percent of GDP. In 1995 and thereafter, the average is 12 percent. A difference of 6 percent of GDP is economically large. To determine whether the difference is statistically significant, we ran the following statistical

test. We ran the regression

$$g_i = \alpha + \beta d_i,$$

where g_i is the real gain in decade i and d_i is a dummy variable, which is 1 for the 1995–2004 decade and 0 for the four decades preceding it (which is when data are available). Our estimate for β is 6.2 percent with a standard error of 2.9 percent, indicating that the change in the mean is statistically significant.

In Figure 10, we plot average real holding gains relative to GDP for the United States along with the extended model's prediction for them. Both curves rise significantly in the late 1990s. The rise is coincident with the dramatic rise in hours.

These results lead us to the conclusion that the model satisfies our prediction criterion.

3.5. Is Success Guaranteed?

A crucial element of the theory is intangible investment, which is directly measurable only in part. We treat the total as unmeasurable and, therefore, unobserved. Does this imply that intangible investment is simply making up for whatever is missing in the standard theory? In this section (and more fully in McGrattan and Prescott 2007), we demonstrate that intangible capital per se does not resolve the puzzling 1990s. If we extend the basic neoclassical model to include intangible capital but assume that technological change is *neutral* then the theory does not satisfy the two criteria we require to successfully resolve the puzzling 1990s boom.

In the alternative version of the model, we introduce a sequence of labor wedges $\{L_{wt}\}$ that are chosen so that the household's intratemporal first-order condition is satisfied,

$$\frac{\psi(1+\tau_{ct}c_t}{1-h_t} = L_{wt}(1-\tau_{ht})w_t,$$

and assume that the TFPs in (3.1) and (3.2) vary proportionally. The wedges are proxies for labor distortions other than government taxes on labor. If the income shares are common in the two activities, as assumed above, the relative price of intangible investment q is constant. We normalize it to one. As above, there are two "free" parameter sequences. In this case, they are $\{A_t^1, L_{wt}\}$.

We find the implications of the alternative model are grossly at odds with what is reasonable behavior for labor distortions and intangible investments. The resulting sequences of $\{L_{wt}\}$ and $\{x_{It}\}$ oscillate wildly. For example, the series for the labor wedge oscillates between 0.8 and 1.4 and displays little persistence. Given what we know about labor markets, this pattern is unreasonable. Furthermore, an hours boom is generated only if TFP and capital tax rates are also oscillatory and offsetting, which is unreasonable in the case of TFP and counterfactual in the case of capital tax rates.

In summary, it is not the inclusion of intangible capital per se that resolves the puzzling U.S. boom in the 1990s. We find that including both intangible capital and non-neutral technological change resolves the puzzle. We turn now to using our theory as a tool for uncovering what actually did happen in the 1990s.

4. Reevaluation of the U.S. Economy in the 1990s

What does all of this mean for U.S. labor productivity and investment? If some output is unmeasured relative to inputs, then GDP and productivity estimates are biased downward. If the mismeasurement is intangible investment, then the investment estimates are also biased downward. Our extended model's predictions for variables with and without intangible investment demonstrate how distorted standard data and models are for assessing the 1990s.

In Figure 11, we compare two predictions for business labor productivity, both computed from the extended model. One is the model's prediction for business value added—without intangible investment included—divided by total business hours. This is what a national accountant would construct. The other includes intangible investment as part of business value added. Both series are detrended by 2 percent annually and set equal to 100 in 1990. Notice how different the predictions are. Measured labor productivity, which is what national accountants would record, shows a significant fall relative to trend up to 1997 and then a sharp increase through 2000. But true productivity, including intangible investment, fell only until 1993 and then, starting in 1994, grew very quickly. Over the

period 1993–2000, the difference in growth rates for these two series is 0.8 percent per year.

In Figure 12, we compare the model's two measures of total investment: tangible investment and tangible plus intangible investment. Again, both are detrended by 2 percent annually and normalized to 100 in 1990. And again, the predictions—with and without intangible investment—are very different. Between 1991 and 1999, tangible investment rose almost 20 percent. Total investment, however, rose more than 30 percent.

In summary, our results show that standard accounting measures and predictions of models without intangible investment do not accurately reflect what was going on in the U.S. economy during the 1990s.

5. Conclusion

We find that non-neutral technological change in the production of intangible investment goods was what gave rise to the puzzling behavior of the U.S. economy in the 1990s. This change resulted in a boom in intangible investment, which is not included in the national accounts measure of output. Once this feature of reality is introduced into the basic neoclassical growth model to obtain what we call the extended growth model, we see that the U.S. economy in the 1990s is in close conformity with theory for both the income and product sides of the national accounts and for the jump in average accrued capital gains. Furthermore, microeconomic observations strongly support our theory of non-neutral technological change.

The important implication of our analysis is that our extended growth model with intangible investments should be used in aggregate economic analyses. Indeed, we see it as a significant improvement on the basic neoclassical growth model.

Appendix A.

Data Appendix

The three main sources for our data are the Bureau of Economic Analysis (BEA), which publishes the national accounts and fixed asset tables; the Federal Reserve Board, which publishes the Flow of Funds tables; and the Bureau of Labor Statistics, which publishes data on hours and population. In this appendix we provide details on the specific data we use and the necessary revisions we make so that the data are consistent with growth theory.

A.1. National Accounts and Fixed Assets

A.1.1. Overview and Sources

Table A contains a summary of the revised national accounts along with averages for the period 1990–2003. The table numbers and sources of the raw data are listed in parentheses. The sources are tables from the BEA's national income and product accounts (NIPA) and fixed asset (FA) tables, and the Federal Reserve's Flow of Funds accounts (FOF). For example, NIPA 7.5 is Table 7.5 from the BEA NIPA tables. The averages for the period we study are relative to GDP and are included to help the reader with the magnitudes of each of the categories.

We have organized Table A as follows. Tables A1 and A2 are the income side of our revised accounts. In Table A1, we display the components of our measure of domestic business value added. This measure is close to, but not exactly the same as, the sum of the value added of corporate business, sole proprietorships and partnerships, and other private business as defined in the NIPA tables. In Table A2, we display the components of our measure of domestic non-business value added. This measure is close to, but not exactly the same as, the sum of value added of the household business sector, nonprofits, general government, and government enterprises. Table A3 provides details of the product side of the accounts along with totals for the income side (for comparison). We have categorized tangible investment into business and non-business as in the case of incomes. That is, investments of corporations and noncorporate business are included with business investment, and investments of household business, nonprofits, and government are included with non-business investment.

Data on capital stocks are used to impute some services of capital when we revise the accounts. They are also used to set certain model parameters and to initialize stocks when computing model equilibria. We use BEA reproducible stocks (FA Table 1.1 for totals and FA Table 6.1 by owner). To that we add land values based on Federal Reserve market values of real estate from balance sheets of households (FOF B100), nonfarm nonfinancial corporations (FOF B102), and nonfarm noncorporate (FOF B103). For farmland, we follow Hansen and Prescott (2002) and assume it is roughly 0.08 times GDP.

A.1.2. Revisions

We now describe how we revise the national accounts to make them consistent with our model. Three adjustments are necessary in the models with or without intangible investment. A fourth adjustment is necessary when we include intangible investment.

Consumption Taxes. Unlike the NIPA, our model output does not include consumption taxes as part of consumption and as part of value added. We thus subtract sales and excise taxes from the NIPA data on taxes on production and imports (line 26, Table A1 and line 24, Table A2) and from personal consumption expenditures (line 10, Table A3) since these taxes primarily affect consumption expenditures. As a result of this adjustment, we use producer prices rather than a mixture of producer and consumer prices.

Financial Services. We treat some of the NIPA's financial services as intermediate rather than as final and, therefore, need to subtract them from GDP and from consumption services. Specifically, we subtract personal business expenses for handling life insurance and pension plans from net interest (line 21, Table A1) and from personal consumption expenditures (line 9, Table A3).

Fixed Asset Expenditures. We treat expenditures on all fixed assets as investment. Thus, spending on consumer durables is treated as an investment rather than as a consumption expenditure and moved from private consumption (line 8, Table A3) to non-business tangible investment (line 22, Table A3). We introduce a consumer durables services sector in much the same way as the NIPA introduces owner-occupied housing services. Households rent the consumer durables to themselves. Specifically, we add depreciation of consumer durables to consumption of fixed capital of households (line 5, Table A2) and to private consumption (line 11, Table A3). We add imputed additional capital services for consumer durables to capital income (line 26, Table A2) and to private consumption (line 12, Table A3). We assume a rate of return equal to 4.1 percent, which is an estimate of the return on other types of capital. A related adjustment is made for government capital. Specifically, we add imputed additional capital services for government capital to capital income (line 27, Table A3) and to public consumption (line 15, Table A3).

Intangible Investment. We introduce intangible investment in the growth model in Section 3. Our output measure includes intangible investment. Thus, total product in the model is the sum of intangible investment and gross domestic product (which we define to be NIPA gross domestic product *after* adjustments are made for consumption taxes, intermediate financial services, and consumer durables). On the income side of our extended model accounts, we add capital gains $q_t x_{It}$. Fraction χ of these gains is "sweat investment," which is allocated to labor income (line 12, Table A1). Fraction $1 - \chi$ of these gains is "expensed investment," which is allocated to capital income (line 27, Table A1). On the product side, we add "business intangible investment" (line 27, Table A3). In Section 3, we describe our calculations.

A.1.3. Tax Rates on Consumption and Labor

We use data from the U.S. national accounts to construct estimates for the tax rates on

consumption and labor.

The tax rate on consumption is found by taking the ratio of sales taxes in NIPA to consumption expenditures in NIPA (which include sales taxes). In our measure of sales taxes, we include federal excise taxes and customs, state and local sales taxes, and other non-property licenses and fees. Our measure of NIPA consumption expenditures includes adjustments for consumer durables. Denoting sales tax by $\tau_c c$ and NIPA consumption expenditures by $c + \tau_c c$, the ratio yields $\tau_c/(1 + \tau_c)$. It is easy to determine τ_c from this ratio.

For the marginal tax rate on labor, we apply essentially the same methodology as in Prescott (2004). Specifically, we take the effective labor tax to be the sum of a marginal income tax rate and a marginal tax rate for social security. The income tax rate is computed as follows. Take personal current taxes in NIPA (which are direct taxes paid by households) and divide them by GDP plus net gain from sale of assets less depreciation and taxes on production and imports. We include gains from asset sales because personal current taxes include taxes on these gains. Prescott (2004) multiplies the income tax rate by 1.6 in order to get estimates of the *marginal* rate comparable to Feenberg and Coutts (1993). The marginal tax rate on social security is computed as follows. Take contributions for government social insurance in NIPA and divide them by labor income. For labor income, we sum compensation and 70 percent of proprietors' income.

A.2. Hours and Population

The primary source of our hours and population data is the U.S. Department of Labor, Bureau of Labor Statistics, *Employment and Earnings*. They are based on the Current Population Survey (CPS). We briefly describe these data here. Full details are given in Prescott, Ueberfeldt, and Cociuba (2005).

The population covered by our series is the total noninstitutional population, ages 16 to 64, for the United States. Prior to 1982, military hours are estimated and added to civilian hours from the CPS. After 1982, they are included in the CPS estimate of total hours.

For versions of the growth model with business and non-business sectors, we also categorize CPS hours as business and non-business. Using the March supplement (through www.ipums.org), we construct business hours as the sum of hours for the self-employed—both incorporated and unincorporated—and hours for private wage and salary workers less hours for employees in nonprofits. Because private wage and salary workers include employees at nonprofits, we use BEA data on compensation in nonprofits, and assuming an average wage rate equal to the economy-wide average, we can infer hours for nonprofits. Hours in the non-business sector are found by subtracting business hours from the total. We use the hours from the March supplement sample to compute the fractions of hours in business and non-business. For our final series, we multiply these fractions by total hours in the monthly CPS sample.

Table A. Revised National Accounts, Averages Relative to GDP, 1990–2003

A1. Domestic Business Value Added

1	Domestic Business Value Added	.748
2	Consumption of fixed capital	.082
3	Corporate business (NIPA 7.5)	.067
4	Sole proprietorships and partnerships (NIPA 7.5)	.013
5	Other private business (NIPA 7.5)	.003
6	Labor income	.483
7	Compensation of employees	.421
8	Corporate business (NIPA 1.13)	.382
9	Sole proprietorships and partnerships (NIPA 1.13)	.036
10	Other private business (NIPA 1.13)	.002
11	70% proprietors' income with IVA and CCadj (NIPA 1.13)	.049
12	Sweat investment (authors' calculations)	.024
13	Capital income	.163
14	Corporate profits with IVA and CCadj (NIPA 1.13)	.073
15	30% proprietors' income with IVA and CCadj (NIPA 1.13)	.021
16	Rental income of persons with CCadj (NIPA 1.13)	.006
17	Net interest and miscellaneous payments	.022
18	Corporate business (NIPA 1.13)	.014
19	Sole proprietorships and partnerships (NIPA 1.13)	.012
20	Other private business (NIPA 1.13)	.005
21	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
22	Taxes on production and imports[b]	.026
23	Corporate business (NIPA 1.13)	.056
24	Sole proprietorships and partnerships (NIPA 1.13)	.008
25	Other private business (NIPA 1.13)	.002
26	*Less*: Sales tax (NIPA 3.5)	.040
27	Expensed investment (authors' calculations)	.024

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A2. Domestic Non-business Value Added

1	Domestic Non-business Value Added	.337
2	Consumption of fixed capital	.099
3	Households	.084
4	Excluding consumer durables (NIPA 7.5)	.012
5	Consumer durable depreciation (FOF F10)	.062
6	Nonprofits (NIPA 7.5)	.004
7	General government (NIPA 7.5)	.018
8	Government enterprises (NIPA 7.5)	.003
9	Labor income	.154
10	Compensation of employees	.154
11	Households (NIPA 1.13)	.001
12	Nonprofits (NIPA 1.13)	.042
13	General government (NIPA 1.13)	.099
14	Government enterprises (NIPA 1.13)	.012
15	Capital income	.083
16	Current surplus of government enterprises (NIPA 1.13)	.001
17	Rental income of persons with CCadj (NIPA 1.13)	.008
18	Net interest and miscellaneous payments	.033
19	Households (NIPA 1.13)	.031
20	Nonprofits (NIPA 1.13)	.002
21	Taxes on production and imports[b]	.004
22	Households (NIPA 1.13)	.011
23	Nonprofits (NIPA 1.13)	.001
24	*Less*: Sales tax (NIPA 3.5)	.007
25	Imputed additional capital services[c]	.038
26	Household, consumer durables	.013
27	Government capital	.025

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A3. Domestic Value Added and Product

1	TOTAL ADJUSTED DOMESTIC INCOME	1.091
2	DOMESTIC BUSINESS VALUE ADDED	.748
3	DOMESTIC NON-BUSINESS VALUE ADDED	.337
4	STATISTICAL DISCREPANCY	.006
5	TOTAL ADJUSTED DOMESTIC PRODUCT	1.091
6	Private consumption	.618
7	Personal consumption expenditures (NIPA 1.1.5)	.678
8	*Less*: Consumer durables (NIPA 1.1.5)	.083
9	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
10	*Less*: Sales tax, nondurables and services (NIPA 3.5)	.042
11	Consumer durable depreciation (FOF F10)	.062
12	Imputed additional capital services[c]	.013
13	Public consumption (NIPA 3.1)	.179
14	Government consumption expenditures (NIPA 3.1)	.154
15	Imputed additional capital services[c]	.025
16	Business tangible investment[d]	.112
17	Corporate gross private domestic investment (FOF F6)	.092
18	Noncorporate gross private domestic investment (FOF F6)	.020
19	Non-business tangible investment	.134
20	Household	.114
21	Excluding consumer durables (FOF F6)	.036
22	Consumer durables (NIPA 1.1.5)	.083
23	*Less:* Sales tax, durables (NIPA 3.5)	.005
24	Nonprofits (FOF F6)	.007
25	Government investment (NIPA 3.1)	.033
26	Net exports of goods and services (NIPA 1.1.5)	−.021
27	Business intangible investment (authors' calculations)	.048

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.
[a] Expense is for handling life insurance and pension plans.
[b] This category includes business transfers and excludes subsidies.
[c] Imputed additional capital services are equal to 4.1 percent times the current-cost net stock of government fixed assets and consumer durables goods (FA 1.1).
[d] 10 percent of farm business is in corporate, with the remainder in noncorporate.

Appendix B.

Parameters

In this appendix, we report and motivate the parameters and exogenous technology and tax series used as inputs for the three models described in the main text. The parameter values are summarized in Table B.1. The exogenous technology and tax rate series are summarized in Table B.2. In a separate technical appendix (McGrattan and Prescott 2007), we also demonstrate, by doing sensitivity analysis, that our main results are robust.

For interest and growth rates, we use estimates based on U.S. trends. In particular, we set the interest rate at 4.1 percent (as in McGrattan and Prescott 2005a) and the annual growth in population η at 1 percent. We also assume that per capita GDP and its components grow at 2 percent annually ($\gamma = .02$). These choices imply that $\beta = .98$. These parameters are used in all versions of our model and ensure that the marginal rate of substitution is the same across experiments.

In all three models, we use constant tax rates on capital. In both versions of the growth model in which we distinguish business and non-business activity, we set the profits tax rate to $\tau_p = 0.35$. because most of the taxes on profits are corporate income taxes. In both cases, we set the distribution tax $\tau_d = 0.15$, which is slightly less than our estimate in earlier work (McGrattan and Prescott 2005a) for corporate distributions; this is appropriate because noncorporate taxes are not taxed twice. In the one-sector version of the growth model, we set the rates lower because most of the capital is not in the business sector and not affected by τ_p or τ_d. In fact we set these rates equal to 0.42 times the rates used in the business sector, since business tangible capital is 42 percent of total tangible capital. For property tax rates, we use NIPA property tax revenues (in "taxes on imports and production") to infer values for τ_k in the one-sector and two-sector versions of the model. Details of our labor and consumption tax rates are provided in Appendix A.

For the remaining parameters, we use 1990 levels of U.S. data to obtain estimates. (See Appendix A for a full description of the data.) Specifically, we use the consumption share of GDP, the tangible investment share of GDP, the total and business tangible capital stocks (including land) as shares of GDP, and hours as a fraction of discretionary time. We use 52 weeks times 100 hours per week as an estimate of discretionary time.

The ratio of tangible investment to the stock is used to infer the rate of depreciation of tangible capital in total or, if there are two sectors, in business. In Table B.1, we report estimates of depreciation for each of the three models. These rates are slightly lower than is typical in the literature because we include land and inventories in our estimates of the capital stock. If we do not include them, the estimates for annual depreciation are on the order of 5 or 6 percent. There is no way to determine δ_I. For this rate, we chose 0 and experimented with other values to make sure our main results did not change.

Given an interest rate, tax rates on capital, and the ratio of tangible capital to output, we can infer the share of tangible capital in producing final goods and services in the models

without intangible investment.[14] For the one-sector version of the model, we find a capital share of 0.34. For the two-sector version, the capital share in business is 0.28. This is lower because the non-business sector is more capital intensive.

In the extended model, we need more information because we do not know the split of tangible capital in final goods production and intangible capital production. Here we use *reported* NIPA compensation for 1990, which, in theory, is equal to total compensation less sweat investment. For our baseline results, the input elasticities for producing both final goods and intangible capital are assumed to be the same (that is, $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$). This restriction along with information on NIPA compensation allows us to determine all capital shares. In Table B.1, we report that the shares for tangible capital in production are 0.26, slightly lower than that for the two-sector model with no intangible investment. The intangible shares are 0.076.

The household's intratemporal condition, along with U.S. observables and estimates of the capital shares and tax rates, implies a value for the utility parameter ψ. In Table B.1, we report the values for each of the models. They are in the range of estimates used in the business cycle literature.

The final parameter to be set is χ, the fraction of intangible investment that is financed by capital owners. This parameter is used only in our extended growth model with intangible investment. As noted earlier, the only real ramification of this choice is for tax payments. But the evidence in Figures 2 and 3 indicates that some investment is being done by both shareholders and workers. We chose $\chi = 0.5$ and then experimented with other values. The main effect of varying χ is a change in the effective tax rates on labor and capital.

[14] An alternative procedure uses information on factor incomes to infer cost shares. However, this procedure is invalid when there are intangible investments because the NIPA accounts do not report the total compensation or profits.

TABLE B.1. Model Parameters

Parameter	Expression	Value
Common Parameters		
Growth in population	η	0.01
Growth in technology	γ	0.02
Discount factor	β	0.98
One-Sector Model, No Intangible Investment		
Utility parameter	ψ	1.48
Depreciation rate	δ	0.031
Capital share	θ	0.34
Tax rate on property	τ_k	0.0073
Tax rate on profits	τ_p	0.15
Tax rate on distributions	τ_d	0.064
Two-Sector Model, No Intangible Investment		
Utility parameter	ψ	1.38
Depreciation rate, business	δ	0.033
Capital share, business	θ	0.28
Tax rate on business property	τ_k	0.014
Tax rate on business profits	τ_p	0.35
Tax rate on business distributions	τ_d	0.15
Extended Model, with Intangible Investment		
Utility parameter	ψ	1.32
Tangible depreciation rate, business	δ_T	0.033
Intangible depreciation rate, business	δ_I	0
Tangible capital share, final goods & services, business	θ_1	0.26
Tangible capital share, intangible investment, business	θ_2	0.26
Intangible capital share, final goods & services, business	ϕ_1	0.076
Intangible capital share, intangible investment, business	ϕ_2	0.076
Tax rate on business property	τ_k	0.014
Tax rate on business profits	τ_p	0.35
Tax rate on business distributions	τ_d	0.15
Fraction of intangible financed by workers	χ	0.5

TABLE B.2. Time Series for Tax Rates and Technology

Year	Tax Rates		Technology Parameters			
			One-Sector	Two-Sector	Extended Model	
t	τ_{ct}	τ_{ht}	Model	Model	A_t^1	A_t^2
1990	6.6	31.1	1.49	1.75	1.66	1.53
1991	6.8	30.7	1.47	1.71	1.60	1.48
1992	6.8	30.3	1.48	1.73	1.60	1.44
1993	6.8	30.3	1.46	1.71	1.60	1.49
1994	7.0	30.7	1.45	1.71	1.63	1.59
1995	6.9	31.2	1.44	1.71	1.64	1.63
1996	6.7	31.9	1.44	1.71	1.67	1.69
1997	6.7	32.5	1.44	1.73	1.69	1.74
1998	6.7	33.3	1.45	1.76	1.73	1.77
1999	6.6	33.4	1.46	1.78	1.76	1.79
2000	6.5	34.3	1.46	1.80	1.76	1.79
2001	6.3	34.7	1.45	1.76	1.73	1.78
2002	6.2	30.8	1.45	1.73	1.60	1.59
2003	6.2	28.9	1.43	1.71	1.56	1.53

Appendix C.

Aggregation in the Extended Model

In this appendix, we develop the aggregation theory underlying the technology of our extended growth model with intangible investment. (See Section 3.)

A business is characterized by the stock of its (unmeasured) intangible capital, K_I. This capital can be used for two activities. One activity produces the composite output of the business Y_b, and the other produces intangible investment goods X_I.

Inputs of (measured) tangible capital K_T^i and hours H^i along with K_I produce an intermediate good Z^i via a standard constant returns to scale neoclassical production function f^i for $i \in \{1, 2\}$. In particular, the production functions are

$$Z^i = (K_T^i)^{\theta_i} K_I^{\phi_i} (H^i)^{1-\theta_i-\phi_i}, \quad i \in \{1, 2\}.$$

The quantity of Y_b produced is $g^1(Z^1)$, and the quantity of X_I produced is $g^2(Z^2)$. The functions g^i are increasing, initially strictly convex, then strictly concave, and they satisfy $g^i(0) = 0$. The slope of the maximal tangent ray from the origin is A^i. The point of tangency is $\hat{Z}^i$. The margin of adjustment is the number of units operated, which is variable. The capital stock K_I can be split over businesses through mergers, acquisitions, and spin-offs. All production units that are operated will have the same K_I. This K_I will depend upon the relative prices of the three inputs. Production units of type i will be operated at level $\hat{Z}^i$ and produce $g^i(\hat{Z}^i)$.

References

Board of Governors of the Federal Reserve System (1945–2005), *Flow of Funds Accounts of the United States*, Statistical Release Z.1 (Washington, DC: Federal Reserve System).

Brynjolfsson, Erik, and Lorin M. Hitt (2000), "Beyond Computation: Information Technology, Organizational Transformation and Business Performance," *Journal of Economic Perspectives*, 14(4): 23–48.

Chang, Yongsung, and Frank Schorfheide (2003), "Labor-Supply Shifts and Economic Fluctuations," *Journal of Monetary Economics*, 50(8): 1751–68.

Chen, Kaiji, Ayşe İmrohoroğlu, and Selahattin İmrohoroğlu (2007), "A Quantitative Assessment of the Decline in the U.S. Saving Rate," Working Paper, USC.

Comin, Diego, and Mark Gertler (2006), "Medium-Term Business Cycles," *American Economic Review*, 96(3): 523–51.

Corrado, Carol A., Charles R. Hulten, and Daniel E. Sichel (2005), "Measuring Capital and Technology: An Expanded Framework", in C. Corrado, J. Haltiwanger, and D. Sichel (eds.), *Measuring Capital in the New Economy* (Chicago: University of Chicago Press).

Corrado, Carol A., Charles R. Hulten, and Daniel E. Sichel (2006), "Intangible Capital and Economic Growth," Finance and Economics Discussion Series, 2006–24.

Couper, Elise A., John P. Hejkal, and Alexander L. Wolman (2003), "Boom and Bust in Telecommunications," *Federal Reserve Bank of Richmond Economic Quarterly*, 89(4): 1–24.

Doms, Mark (2004), "The Boom and Bust in Information Technology Investment," *FRBSF Economic Review 2004*, Federal Reserve Bank of San Francisco (http://www.frbsf.org).

Feenberg, Daniel, and Elisabeth Coutts (1993), "An Introduction to the TAXSIM Model," *Journal of Policy Analysis and Management*, 12(1): 189–94.

Galí, Jordi (2005), "Trends in Hours, Balanced Growth and the Role of Technology in the Business Cycle," *Federal Reserve Bank of St. Louis Review*, 87(4): 459–86.

Galí, Jordi, Mark Gertler, and J. David López-Salido (2007), "Markups, Gaps and the Welfare Costs of Business Cycles," *Review of Economics and Statistics*, 89(1): 44–59.

Greenwood Jeremy, Zvi Hercowitz, and Per Krusell (1997), "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87(3): 342–62.

Hall, Robert E. (1997), "Macroeconomic Fluctuations and the Allocation of Time," *Journal of Labor Economics*, 15(1): S223–50.

Hansen, Gary D., and Edward C. Prescott (2002), "Malthus to Solow," *American Economic Review*, 92(4): 1205–17.

Hornstein, Andreas and Per Krusell (1996), "Can Technology Improvements Cause Productivity Slowdowns?" *NBER Macroeconomics Annual*, Vol. 11, 209–59. (Cambridge,

MA: MIT Press).

Ireland, Peter N., and Scott Schuh (2007), "Productivity and U.S. Macroeconomic Performance: Interpreting the Past and Predicting the Future with a Two-Sector Real Business Cycle Model," Working Paper 642, Boston College.

Kahn, James A., and Robert W. Rich (2007), "Tracking the New Economy: Using Growth Theory to Detect Changes in Trend Productivity, *Journal of Monetary Economics*, 54(6): 1670–1701.

McGrattan, Ellen R., and Edward C. Prescott (2005a), "Taxes, Regulations, and the Value of U.S. and U.K. Corporations," *Review of Economic Studies*, 72(3): 767–96.

McGrattan, Ellen R., and Edward C. Prescott (2005b), "Productivity and the Post-1990 U.S. Economy," *Federal Reserve Bank of St. Louis Review*, 87(4): 537–49.

McGrattan, Ellen R., and Edward C. Prescott (2007), "Technical Appendix: Unmeasured Investment and the Puzzling U.S. Boom in the 1990s," Staff Report 395, Federal Reserve Bank of Minneapolis.

Miller, Merton H. (1977), "Debt and Taxes," *Journal of Finance*, 32(2): 261–75.

National Science Foundation, Division of Science Resources Statistics (1953–2003), *National Patterns of Research and Development Resources* (Arlington, VA).

Powell, Michael K. (2002), "Remarks at the Thomas Weisel Partners, Growth Forum 4.0," June 17 (Santa Barbara, CA: www.fcc.gov/Speeches).

Prescott, Edward C. (2004), "Why Do Americans Work So Much More Than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review*, 28(1): 2–12.

Prescott, Edward C., Alexander Ueberfeldt, and Simona Cociuba (2005), "U.S. Hours and Productivity Behavior Using CPS Hours Worked Data: 1959-I to 2005-II," Manuscript, Research Department, Federal Reserve Bank of Minneapolis.

Ritter, Jay R., and Ivo Welch (2002), "A Review of IPO Activity, Pricing, and Allocations," *Journal of Finance*, 57(4): 1795–1828.

Ruggles, Steven, et al. (2004), Integrated Public Use Microdata Series: Version 3.0 [Machine-readable database] (Minneapolis, MN: Minnesota Population Center) (http://www.ipums.org).

Smets, Frank, and Rafael Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3): 586–606.

Solow, Robert M. (1987), "We'd Better Watch Out," *New York Times Book Review*, July 12, p. 36.

U.S. Department of Commerce, Bureau of Economic Analysis (1929–2005), National Income and Product Accounts, *Survey of Current Business* (Washington, DC: U.S. Government Printing Office).

Figure 1

U.S. and Basic Model Per Capita Hours Worked

Annual, 1990=100, 1990–2003

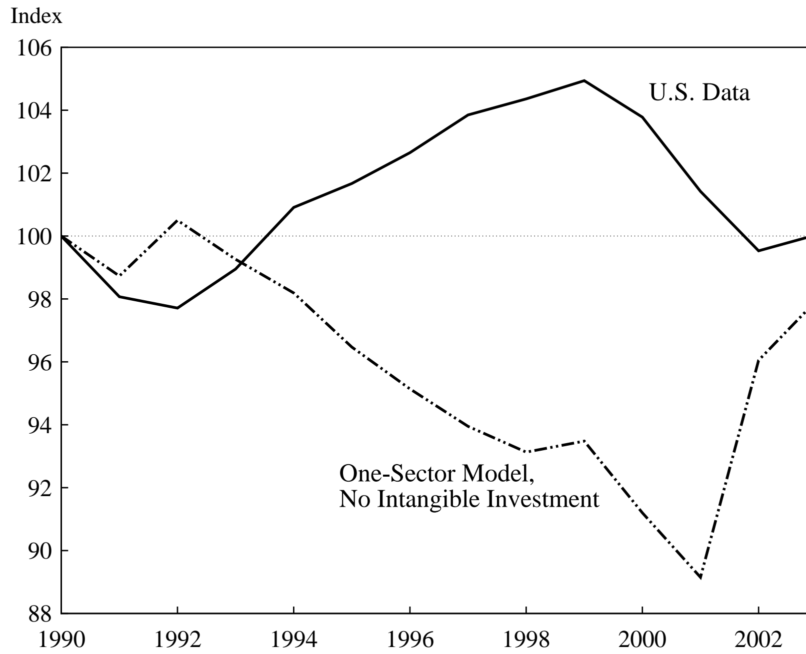

Figure 2

U.S. and Basic Model Per Capita Real GDP

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

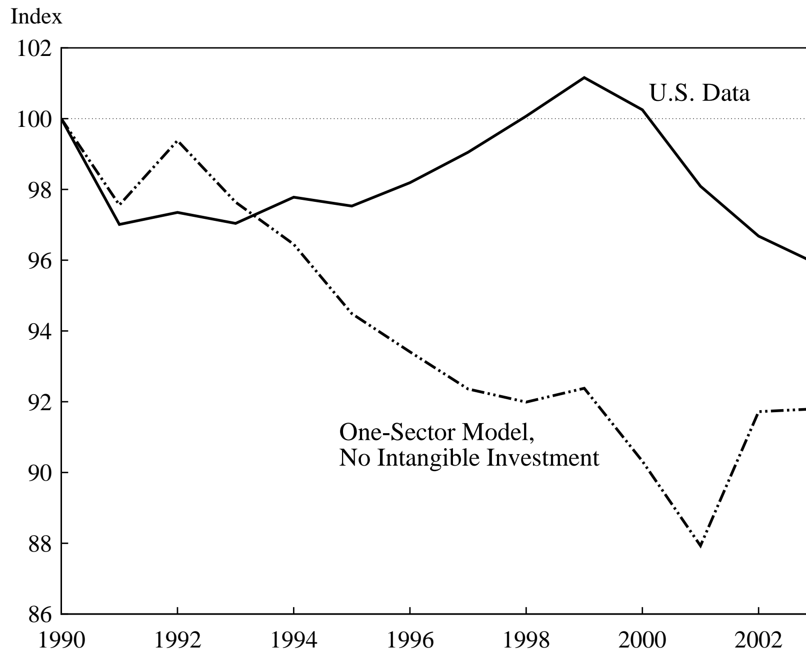

Figure 3

Average Weekly Hours Worked by Noninstitutional Population, Age 16–64, and NIPA Real Compensation Per Hour

Annual, Compensation Divided by 1.02^t, 1990=100, 1990–2003

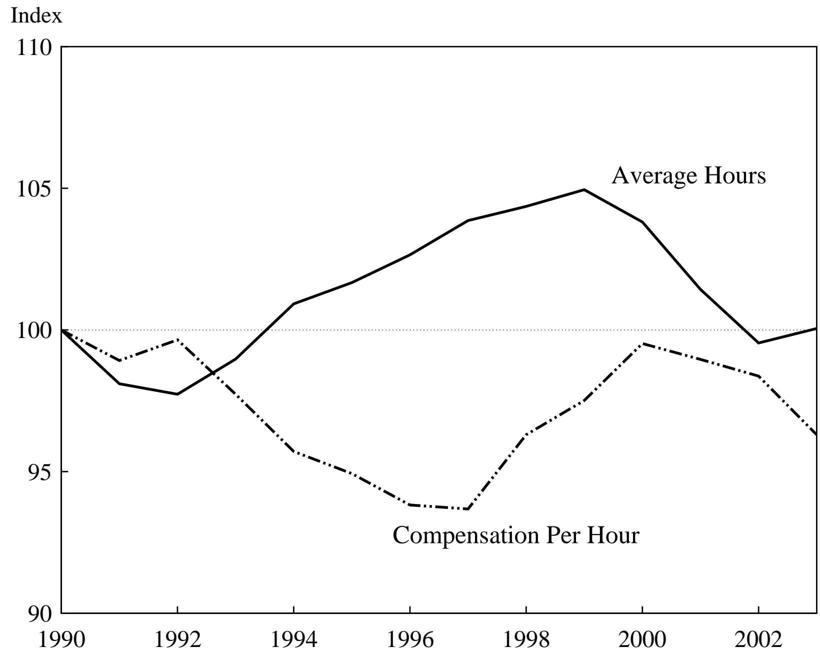

Figure 4

NIPA Real Per Capita GDP and Corporate Profits

Annual, Series Divided by 1.02^t, 1990–2003

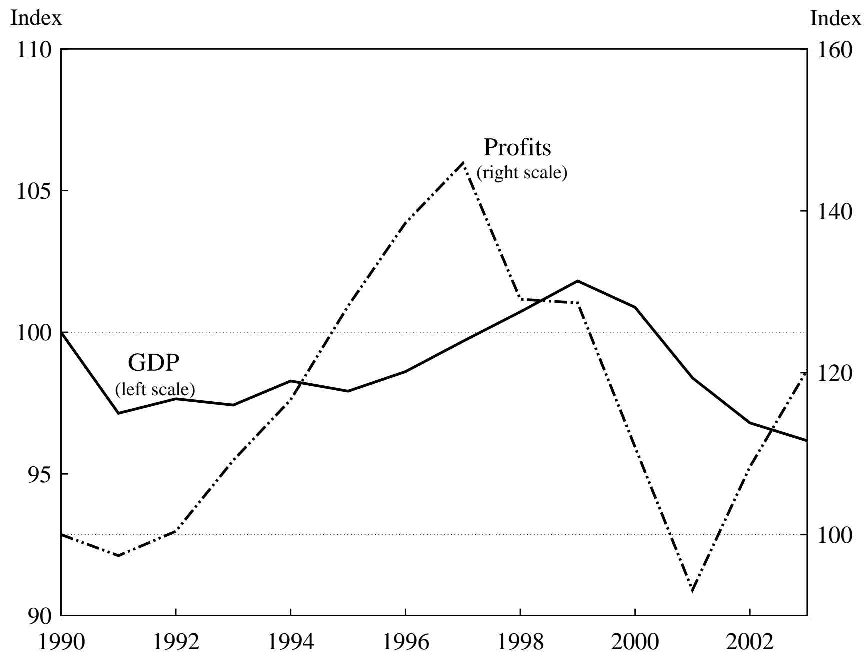

Figure 5

U.S. and Extended Model Real Total Factor Productivity
Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

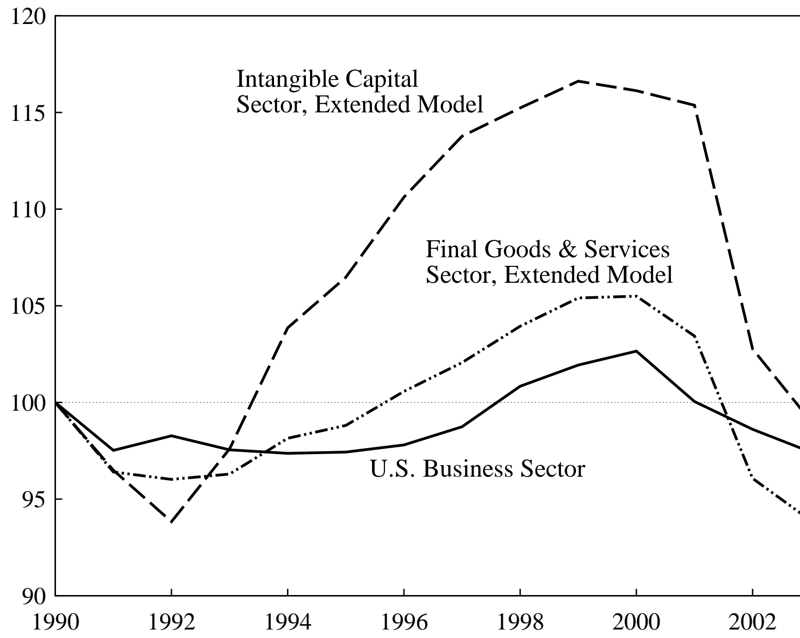

Figure 6

Extended Model Intangible Share of Total Output
Annual, 1990–2003

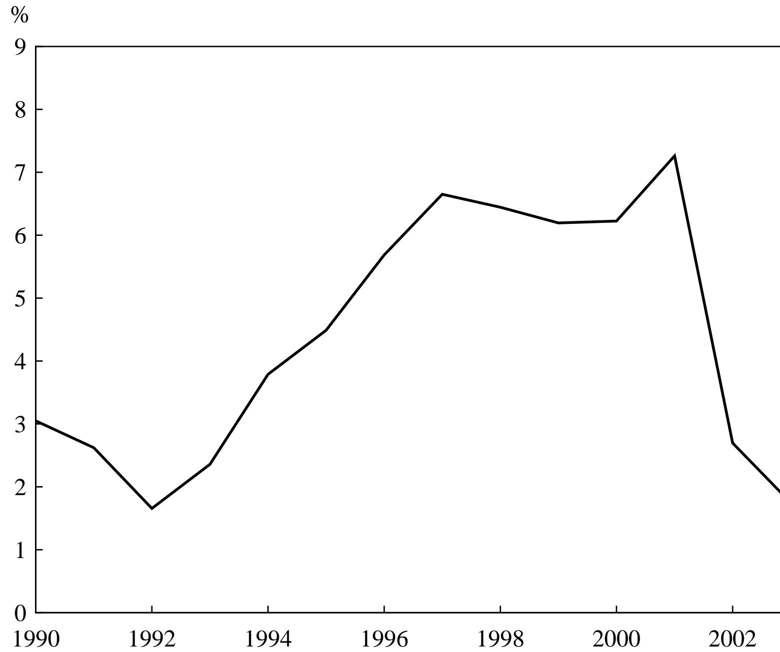

Figure 7

U.S. and Extended Model Per Capita Hours Worked
Annual, 1990=100, 1990–2003

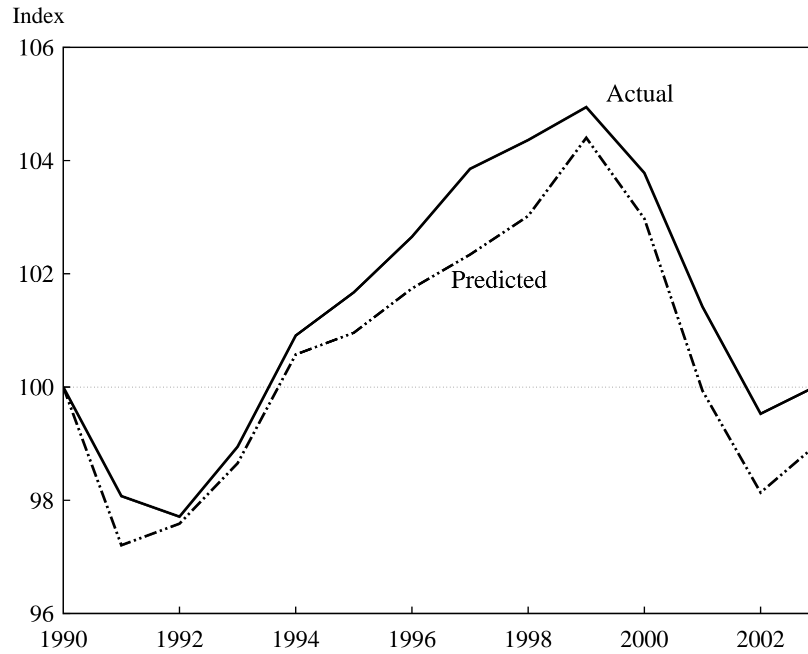

Figure 8

U.S. and Extended Model Per Capita Real GDP
Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

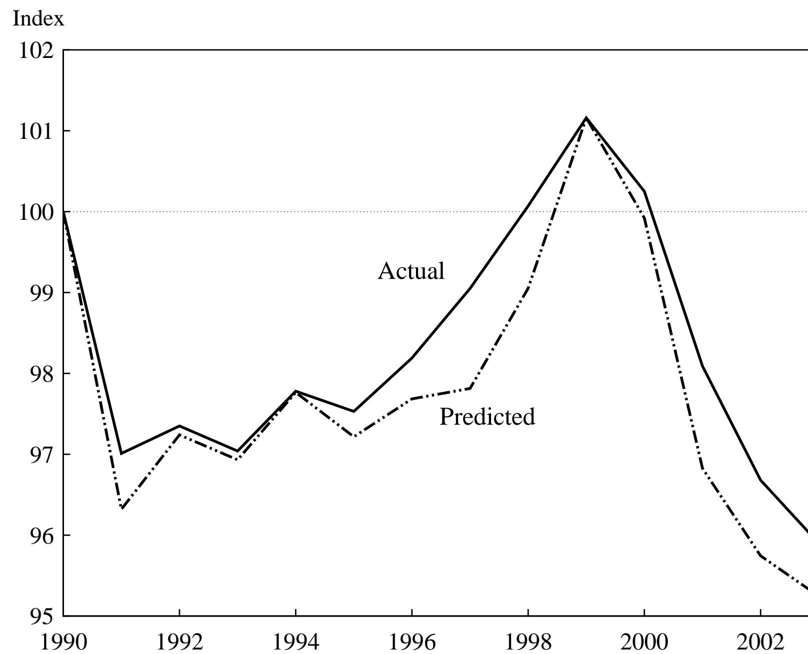

Figure 9

U.S. and Extended Model Real Business Compensation Less Sweat

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

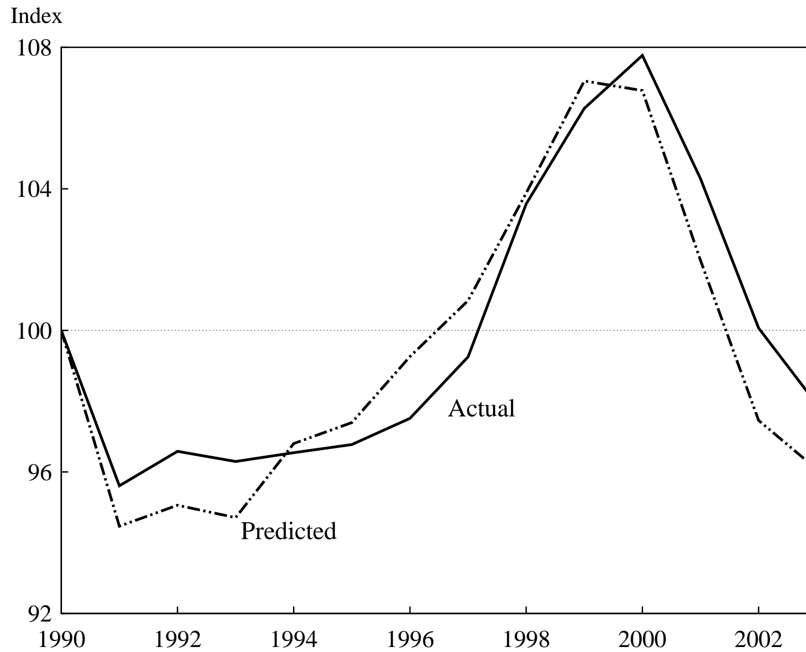

Figure 10

U.S. and Extended Model Real Holding Gains

Annual, % of Real GDP, Excluding Real Estate, 1990–2003

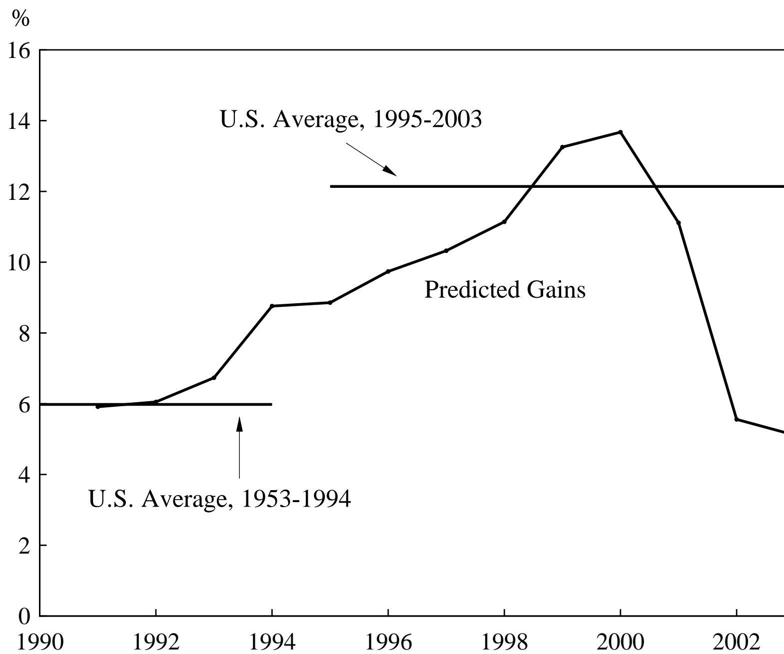

Figure 11

Extended Model Real Business Productivity

Annual, Series Divided by 1.02^t, 1990=100, 1990–2003

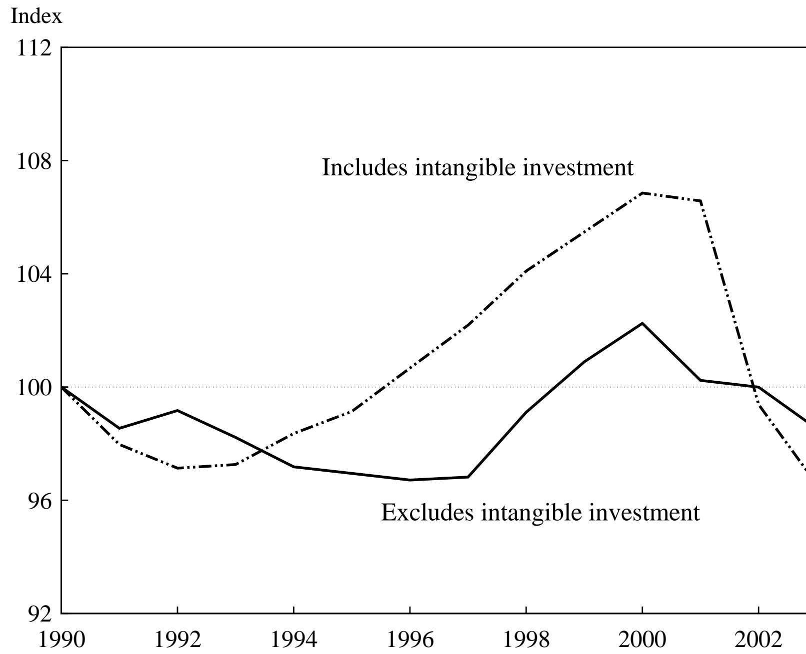

Figure 12

Extended Model Real Per Capita Investment

Annual, Series Divided by 1.02^t, 1990=100 1990–2003