

# Welfare Effect of a Price Change: The Case of Lotto 6/49\*

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May 5, 2008

## Abstract

On June 2, 2004 Lotto 6/49 increased its ticket price from one dollar to two dollar. The price change provides a unique opportunity to study the welfare effect on a price change of a product involving risk. Using an explicit utility theory on lottery first developed by Erwin Diewert and extended by Kam Yu, we estimate the welfare effect and the real price change of the nominal price increase. Initial results indicate that the ticket price change, with some minor rule changes in the prize structure, have improved the quality of the product. Simply put, the higher jackpot prizes provide more excitement for the lottery players. Consumer welfare has increased and the real price of playing the game has decreased. As a result productivity of the lottery sector has also improved.

JEL Classification: C43, D81, E01, L83

Keywords: system of national accounts, direct measurement, volume index, quantity index, choice under risk, lotteries, gambling, non-expected utility theory.

## 1 Introduction

Lotto 6/49 has been played weekly across Canada since June 12, 1982. For example, the average Canadian household spends \$154 in 2005 on government-run lotteries

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\*Paper presented to the 2008 World Congress on National Accounts and Economic Performance Measures for Nations, May 12–17, 2008, Washington, D.C.

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(Statistics Canada, 2005).<sup>1</sup> Evidently, the lottery industry is a significant part of the gross domestic product. In this paper we analyze the innovation of rules of Lotto 6/49 that was executed on June 2, 2004, study the output and the price index of Lotto 6/49 before and after June 2, 2004, and calculate the price elasticity of demand for lottery tickets.

Instead of the traditional expected-utility model, we use a new economic approach to estimate the real output and price of the Lotto 6/49. In economic theory the expected utility hypothesis is well known and widely applied in solving problems with uncertainty. However, the traditional expected-utility hypothesis (EUH) and expected-utility models are not sufficiently flexible to provide an adequate empirical description of economic behavior in a risky environment (Epstein, 1992). In gambling, according to the EUH a risk-averse, consumer will never buy a lottery ticket, since the game is unfair. Therefore, the implication of the EUH cannot accommodate the fact that consumers are risk-averse in buying insurance and at the same time risk-seeking in playing games of chance. To resolve the paradox, Diewert (1993, 1995) develops a simple 2-state model of gambling using the implicit expected utility theory. The model has been expanded by Yu (2004) to a  $N$ -state model with unequal probabilities to calculate the real output and price of the lottery sector. In this paper we update Yu's model according to the new Lotto 6/49 rules and study the impact of the rule change.

The structure of the paper is as follows. Section 2 briefly reviews the background of the game of lottery and compares the rules of the Canadian Lotto 6/49 before and after June 2, 2004. A comparison with Super 7, another national lottery game, is also provided. In Section 3 we introduce Diewert's 2-state model and expand Yu's  $N$ -outcome model. Section 4 reports the data sources and the estimates of the real output, the price index and the elasticity of the tickets demand. Finally, Section 5 discusses and concludes our findings.

## 2 Lottery Background

### 2.1 History

Lotteries have been successful in many countries for hundreds of years. It existed at least before Christ. The Roman emperor Augustus Caesar once used lotteries to fund public works (Ziemba, 1986). Germany had its first city lottery in 1470 and it has had a national lottery game for over 40 years. Italian national lottery has operated continuously since 1530. United Kingdom first organized its lottery in 1566 aimed to raise fund to fight the French (Walker, 1998). At present over

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<sup>1</sup>Marshall (1998) reports that consumers often underreport their spending in gambling by a factor of three.

100 countries run lotteries. Spain, Ireland, Canada, the UK, USA and several Australian have the largest lotteries in the world. Most of these countries have several lottery games throughout the countries or provinces. These lottery games are often owned by syndicates, and the revenues are shared by the companies, the retailers and the government or institution for some good causes such as health care, education, environment and community charity. In Canada, lottery was illegal until 1969. Canada's first major national lottery was organized for the Montreal Olympic Games in 1973. Now, the two lottery games, Lotto 6/49 and Super 7 are played across the entire country. They are operated by five provincial lottery organizations — British Columbia Lottery Corporation, Western Canada Lottery Corporation, Ontario Lottery Corporation, Lotto Quebec, and Atlantic lottery Corporation. Each company also offers smaller lottery games that occur across one or two provinces. Compared with the national lotteries, these games usually have lower price ticket and smaller fixed prizes.

## 2.2 Lotto 6/49 in Canada

The Lotto 6/49 has been played throughout Canada since its inception. Customers choose 6 numbers out of the 49 numbers. There are several prizes depending on the total numbers matched. The total amount wagered and take out rate, which is the rate of the amount that allocate to the government, retailers and the corporations, determine the prize fund of the draw according to the following formula:

$$\text{Prize fund} = (1 - \text{takeout rate}) \times \text{total wager}.$$

Except for the smaller prizes, which have fixed rewards, winners will share the prize reward from the pool fund. The pool fund is determined by deducting the prize fund by the total amount of fixed prizes. For each draw, if there is no jackpot winner, the jackpot pool will be passed on to the jackpot pool of the next draw (the “Rollover”). At the beginning, Lotto 6/49 in Canada was similar to lottery in other countries with one-dollar ticket price and one draw per week. Then, as most countries did, Lotto 6/49 changed its draw to twice a week on September 11, 1985, while keeping everything else unchanged. On June 2, 2004, it increased the ticket price from one dollar to two dollars, redesigned the take out rate and shares of the pool fund and added a sixth prize. The organizers claimed that with the changes, customers will have a chance at bigger jackpots more often and better overall odds of winning. Table 1 lists the difference between the two rules.

From Table 1 we see that the share of jackpot has been raised from 50% to 80.5% of the pool fund and the take-out rate has been reduced from 55% to 53%, the average jackpot will be higher than before. The shares of other prizes, which have larger chances to win, have been reduced sharply, while the Jackpot, which has an extremely low chance to win, has been increased. The probability of a rollover is

given by

$$p = (1 - \pi)^{s/v},$$

where  $\pi$  is the probability of winning the jackpot per ticket,  $s$  is total sales and  $v$  is the price of the tickets. The probability falls with  $s$ , since the more tickets sold, the more likely of a matching ticket having been sold to claim the jackpot, and the probability falls with  $\pi$ . On the other hand, the probability of a rollover rises with the ticket price, since the higher the price is, for the fixed sales, the fewer tickets are issued. So as the ticket price increases to two dollar, the chances to have rollovers, double rollovers or multiple rollovers are higher. In terms of odds, it seems that the overall odd has increased because of the new 6th prize. The odds, however, are calculated on a per ticket basis, which is now two dollar. If we consider the odds per dollar, which is 1/64, the overall odd is lower than before.

Also, in the new game there is a rule that if the rollover exceeds \$30 million, then the percentages of the pool fund applying to Jackpot, 2nd prize, 3rd prize, and 4th prize will be changed to 40%, 16%, 15% and 29%, respectively. Nevertheless, up to now rollovers have never exceeded \$30 million.

### 3 Output of Lotto 6/49

In economic theory, the classical expected utility model has been applied to many areas but sometimes the observed behaviour in a risky environment and experimental results contradict the theory. The expected utility theories represent consumers with second order risk aversion rather than first order risk aversion, which imply that the risk premium is proportional to the variance of the gamble but not to the standard deviation of the gamble (Segal and Spivak, 1990). Here we review the newly developed theory in uncertain environment and its application in simple gambling sector.

Based on the the classical theory by von Neumann and Morgenstern (1953), a contingent commodity approach was developed by Arrow (1964). Blackorby, Davidson and Donaldson (1977) place a separability assumption on the contingent preference function and by using the representation theorem proposed by Gorman (1968), they deduce an additively separable functional form as follows:

$$M(x_1, x_2, \dots, x_N) = \phi^{-1} \left[ \sum_{i=1}^N (1/N) \phi(x_i) \right],$$

where  $M$  is called a stochastic preference function that expresses the decision maker's attitude toward risk,  $x_i$  is a certainty utility which can be expressed as

$$x_i = f(y^i), \quad i = 1, \dots, N,$$

$y^i$  denotes the consumption vector of the decision maker if state  $i$  occurs, and  $f$  is the certainty utility function. There are  $N$  states of nature and each state has probability  $1/N$  and  $\phi$  is a continuous, increasing function of one variable. Diewert (1993) derives an implicit expected-utility theorem based on this contingent commodity approach. It can be expressed as

$$\sum_{i=1}^N p_i \phi_u(x_i) - \phi_u(u) = 0,$$

where  $p_i$  is the probability and  $u$  is the overall utility. In this formulation,  $\phi$  is function of the certainty utility of state  $i$ ,  $x_i$  and the overall utility  $u$ . In order to aggregate across consumers, a homothetic preference structure is assumed. Then the above implicit utility function can be written as

$$\sum_{i=1}^N p_i \gamma(x_i/u) - \gamma(1) = 0, \quad (1)$$

where  $\gamma$  is an increasing and continuous function of one variable.

### 3.1 A Simple Model

Using (1), Diewert (1995) derives simple models for measuring the real outputs of some problems in choice under risk. Here we review the simple model in 2-state game and in the next section extend it to a 7-state model for Lotto 6/49. A consumer has a probability  $p_1$  to lose the wager  $w$ , and a probability  $p_2 = 1 - p_1$  to win  $Rw$ , where  $R$  is the payout ratio, which means the ratio of the total wagers allocate to the prize. The outcomes can be written as

$$x_1 = y - w, \quad x_2 = y + Rw, \quad (2)$$

where  $y$  is consumer's real income. Assuming homothetic preferences, the implicit utility function  $\phi_u$  can be written as:

$$\phi_u(z) = \gamma(z/u).$$

To provide a kink in the difference curve, the following functional form for  $\gamma$  is employed:

$$\gamma(z) = \begin{cases} \alpha + (1 - \alpha)z^\beta, & z \geq 1 \\ 1 - \alpha + \alpha z^\beta, & z < 1 \end{cases} \quad (3)$$

where  $0 < \alpha < 1/2, \beta < 1, \beta \neq 0$ . Putting together (1), (2), and (3), the utility function becomes

$$u = [\delta(y - w)^\beta + (1 - \delta)(y + Rw)^\beta]^{1/\beta} \quad (4)$$

where  $\delta \equiv p_1\alpha/[p_1 + (1 - p_1)(1 - \alpha)]$ . Maximizing the utility equation in (4) subject to the constraint  $0 \leq w \leq y$  we can get the optimal wager:<sup>2</sup>

$$w^* = \frac{y(b - 1)}{b + R}$$

where

$$b = \left[ \frac{(1 - p_1)(1 - \alpha)R}{p_1\alpha} \right]^{\frac{1}{1-\beta}}.$$

With observations in  $w^*$  and  $y$ , the parameters  $\alpha$  and  $\beta$  can be estimated using a regression model. Then consumers' utility levels with gambling ( $u^*$ ) and without gambling ( $u^0$ ) can be calculated using (4). The real output of the 2-state game can be defined as

$$Q = u^* - u^0.$$

### 3.2 Model for Lotto 6/49

As mentioned above, Lotto 6/49 had 5 prizes when it was one dollar per ticket and now it has 6 prizes. We now expand the 2-state model to a multiple-state model.

To allow for the different prices per ticket in the two rules, we derive the results using the number of tickets instead of the wager. First we formally define the variables used:

$w$	=	wager,	
$n$	=	number of tickets purchased per draw,	
$v$	=	price per ticket,	
$\pi_i$	=	probability of winning the $i$ -th prize for one single ticket,	$i = 1, \dots, 6$
$p_i$	=	total probability of winning the $i$ -th prize,	$i = 1, \dots, 6$
$p_7$	=	probability of not winning any prize,	
$x_i$	=	state contingent consumption,	$i = 1, \dots, 7$
$y$	=	real disposable income,	
$R_i$	=	payout for the $i$ -th prize, ( $R_7 = 0$ )	$i = 1, \dots, 7,$

where  $n = w/v$  and  $p_i = n\pi_i$ , since buying more tickets will increase the chance to win. Table 4 lists all the probabilities of winning for a single ticket. The probability of not winning any prize is one minus the total probability of winning, that is:

$$p_7 = 1 - \sum_{i=1}^6 p_i = 1 - n \sum_{i=1}^6 \pi_i$$

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<sup>2</sup>The constraint is normally non-binding.

and  $x_i = y + R_i - vn, i = 1, \dots, 7$ . In the one-dollar ticket game  $v = 1$  and therefore  $w = n$ . Also, there are 6 outcomes instead of 7. See Yu (2004) for details.

As we mentioned above, if the consumer has a homothetic preference, his/her contingent preference function can be expressed by (1). Using  $\gamma$  assumed in (3), it becomes

$$\sum_{i=1}^6 p_i \left[ \alpha + (1 - \alpha) \left( \frac{x_i}{u} \right)^\beta \right] + p_7 \left[ 1 - \alpha + \alpha \left( \frac{x_7}{u} \right)^\beta \right] - 1 = 0,$$

or

$$\begin{aligned} \sum_{i=1}^6 n\pi_i \left[ \alpha + (1 - \alpha) \left( \frac{y + R_i - vn}{u} \right)^\beta \right] + \\ \left( 1 - \sum_{i=1}^6 n\pi_i \right) \left[ 1 - \alpha + \alpha \left( \frac{y - vn}{u} \right)^\beta \right] - 1 = 0. \end{aligned}$$

Solving for  $u$ , we get

$$u(n) = \left[ \frac{(1 - \alpha)n \sum_{i=1}^6 \pi_i (y + R_i - vn)^\beta + \alpha (1 - n \sum_{i=1}^6 \pi_i) (y - vn)^\beta}{\alpha + (1 - 2\alpha)n \sum_{i=1}^6 \pi_i} \right]^{1/\beta}. \quad (5)$$

The consumer's problem is to maximize this utility function with respect to  $n$  subject to the constraint  $0 \leq 2n \leq y$ . For notational convenience, we define the following variables as

$$\begin{aligned} d &= y - vn, \\ p &= \sum_{i=1}^6 \pi_i, \\ q &= \sum_{i=1}^6 \pi_i (y + R_i - vn)^{\beta-1}, \\ m &= \sum_{i=1}^6 \pi_i (y + R_i - vn)^\beta. \end{aligned}$$

Notice that although  $w = vn$  appears on the right hand side of the above equations, compared with disposable income  $y$  and payout prize  $R_i$ , which are much more higher than  $w$ , the effects of  $w$  are so trifling that they can be ignored. The first order condition is

$$\begin{aligned} \alpha(1 - \alpha)m - v\beta q(1 - \alpha)n[\alpha + (1 - 2\alpha)np] - \\ v\alpha\beta(1 - np)[\alpha + (1 - 2\alpha)np]d^{\beta-1} - \alpha p d^\beta (1 - \alpha) = 0. \end{aligned}$$

Rearranging as a quadratic equation in  $n$ , we have

$$\begin{aligned} & \{v\beta p(1-2\alpha)[\alpha p d^{\beta-1} - q(1-\alpha)]\}n^2 \\ & + \{v\alpha\beta[\alpha p d^{\beta-1} - (1-2\alpha)p d^{\beta-1} - q(1-\alpha)]\}n \\ & + \alpha[(1-\alpha)m - v\alpha\beta d^{\beta-1} - p d^{\beta}(1-\alpha)] = 0. \end{aligned} \quad (6)$$

Writing (6) as  $an^2 + bn + c = 0$ , where

$$\begin{aligned} a &= v\beta p(1-2\alpha)[\alpha p d^{\beta-1} - q(1-\alpha)], \\ b &= v\alpha\beta[\alpha p d^{\beta-1} - (1-2\alpha)p d^{\beta-1} - q(1-\alpha)], \\ c &= \alpha[(1-\alpha)m - v\alpha\beta d^{\beta-1} - p d^{\beta}(1-\alpha)], \end{aligned}$$

the solution for this quadratic equation is the optimal level of wager

$$n^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We use this equation to estimate the values of parameters  $\alpha$  and  $\beta$ . With the estimated parameters, we can calculate the output of services provided by Lotto 6/49, which is

$$Q^t = u(n^t v^t) - u(0),$$

where  $n^t$  is the number of tickets sold and  $v^t$  is the ticket price in period  $t$ . An implicit price level can be estimated as a sub-index of the CPI by following equation:

$$P^t = w^t / Q^t.$$

## 4 Empirical Analysis

### 4.1 Data Sources and Statistics

Our data on the winning numbers, payout prize and sales of Lotto 6/49 are from January 3, 1998 to May 29, 2004 for the one dollar game and from June 2, 2004 to Aug 30, 2006 with the two-dollar game. The data is partly collected from Lottery Canada web site and partly provided by the company. Monthly data on the CPI and annual data on the number of households, personal disposable income, and participation rates in government lotteries are available from Statistics Canada. The sales volume of each draw is divided by the number of participating households, which gives the average wager per participating household,  $w_t$ . The average personal disposable income per household, adjusted by the CPI, is used as a proxy for  $y_t$ .

Figure 1 depicts the numbers of tickets sold from January 3, 2001 to August 30, 2006. The numbers of tickets sold fell sharply after the ticket price increased to two



dollars. The average number of tickets sold in the one-dollar game in the sample period is 15.93 million while in the two-dollar game it is only 9.29 million. Since the ticket price is double, overall revenue of Lotto 6/49 still increased. Figure 2 plots the total sales revenue over time. After the price increase, the highest sales point is 99,474,164 on October 26, 2005, more than twice the highest sales point in the one dollar period. The interesting point is that the lowest sales volume during the two dollars period was 1,607,940 on August 27, 2005, less than one sixth of the lowest sales point in the one dollar period. However, the jackpot was \$8.45 million, which was five times higher than the total ticket sales of that draw and higher than the average jackpot during the period of the two dollars. In fact at that draw the revenue from sales could only cover the payout of the 5th and 6th prizes.

In the two-dollar game, the increased wagers and higher jackpot rate give gamblers more attractive prize pay-offs. Moreover, fewer tickets sold means that the chance to share prizes with others decreases. The lottery dream with the same winning chance but a higher reward and lower possibility of sharing makes gamblers have a higher expected value for the Lotto 6/49.

Apart from the publicity and attention drawn to the jackpot, how do the pay-offs of other prizes fare? Table 2 compares the average sales, pool fund, number of winners and payouts of prizes of the one dollar with those of the two dollars and their average changes. The biggest jackpot during the sample period of one-dollar ticket was \$15 million, won by a single ticket on September 30, 2000. The biggest jackpot of the two-dollars sample period was \$54 million, won by a single ticket on October 26, 2005. We can see that the percentage increase in take-out (15.2%) is lower than the percentage increase in sales (19.59%), which reflects the lower take-out rate in the new rule. The new game gives a much higher average jackpot than old one while other prizes have decreased in size. This mirrors the higher share rate in the jackpot and lower share rate in the other prizes in the two-dollar game. Clearly, the only thing become more charming under new rule is the size of the jackpot.

## 4.2 Lotto 6/49 and Super 7

Lotto 6/ 49 and Super 7 are both national lottery games in Canada. Compared with Lotto 6/49, which selects six numbers from 1 to 49, Super 7 allow customer to pick seven numbers from 1 to 47. The chance to win any prize in Super 7 is 1 in 17. The jackpot's share of pool fund was 60% before January, 20, 2006 and 73% after, while the corresponding share in the one-dollar Lotto 6/49 game was 50%. The history of Super 7 is shorter than Lotto 6/49, but Super 7 developed rapidly and now it enjoys the same market share as Lotto 6/49. Figure 3 compares the sales of Lotto 6/49 and Super 7 and Table 3 shows the regression analysis of the total sales against time.

From the figure and the table, we can see that before the rule change, there was a downward trend on sales of Lotto 6/49. The rate of increase in total sales of Super 7

is higher than that of Lotto 6/49. After the rule change the rate of increase of Lotto 6/49 has risen but still lower than Super 7's rate. In fact, after the rule change, Lotto 6/49 and Super 7 have become similar in structure: both have the two-dollar price, higher jackpot rate and similar chances to win. The figure shows that after the rule changes, sales in Super 7 and Lotto 6/49 are almost the same. Considering that both Super 7 and Lotto 6/49 are owned by the same corporation, and the similar rules make them more like substitute goods, perhaps the corporation can consider the probability of combining the two games to make a more attractive game.

### 4.3 Estimation and Results

By using the maximum likelihood method, we get the estimated values of  $\alpha$  and  $\beta$ . In the one-dollar ticket period,  $\alpha = 0.1201$  with standard error of  $0.834 \times 10^{-5}$  and  $t$ -ratio of 14,408 and  $\beta = -31.95$  with standard error of  $-0.269 \times 10^{-3}$  and  $t$ -ratio of -118,670. In the two-dollar ticket period,  $\alpha = 0.0319$  with standard error of  $0.3801 \times 10^{-6}$  and  $t$ -ratio of 8,389 and  $\beta = -0.000338$  with standard error of  $0.4312 \times 10^{-3}$  and  $t$ -ratio of -0.78464. Using the estimated value, we can get the money metric utility and the real output of the Lotto 6/49 for each draw in two periods.

Figure 4 illustrates the changes of customers expected utility values in the two periods. Our interpretation is that the new game has increased the expectation and excitement of the players, resulting in higher utility or output. The average output in the two-dollar game is \$26.1 million, which is six times of the average output in the one-dollar-game. The average official GDP at factor cost, which is 15.4% of the revenue (7.1% in retailer commissions and bonuses and 8.3% in other operating expenses) is \$2.73 million. It is one ninth of the average output calculated by the economic approach. Using the official statistics as the total input cost in the production of the Lotto 6/49 services, we can calculate the games productivity as shown in Figure 5. The average productivity in the two-dollar game is 10.3 and the one-dollar game 1.9. It seems that the innovation of Lotto 6/49 is highly successful. On the producer side the corporations gain more revenue by increasing the price. On the consumer side players now have a more exciting game. Overall the efficiency of production has improved dramatically.

Figure 6 is calculated with the price of January 1, 2004 as the basis price. Contrast with the real output, the price index decreases tremendously after the new rule applied and the average price index in the two-dollar game is 0.26, which is less than one sixth of the average price index in one-dollar game. The price trend in Figure 6 correlates in an opposite way with the changes of productivity in Figure 5. Since  $P = w/Q$ , when wagers do not change too much, the increase in  $Q$  causes the real price to decrease.

We also estimate the elasticity of the demand for the lottery using a simple

formulation:

$$\log Q = \eta \log P + \epsilon \log y + T.$$

where  $\eta$  is the price elasticity of demand,  $\epsilon$  is the income elasticity, and  $T$  is a trend variable to capture taste change over time. The price elasticities of the Lotto 6/49 in the two periods are  $-0.72$  and  $-0.21$  respectively. The absolute value of  $\eta$  decreases with the ticket price increase, that is, the demand for Lotto 6/49 become more inelasticity in the two-dollar game. In one-dollar game the income elasticity of Lotto 6/49 is 2.1, which means that lotto tickets are a “luxury” goods, while in the two-dollar game the income elasticity is 0.76, which means that the lotto tickets are a normal good.

The above results seems paradoxical. In the one-dollar game, the real price is in the inelastic range. Therefore a revenue maximizing firm should increase the price, which is exactly what the lottery corporations do. What is interesting is that they can only control the *ticket* price, not the real price. With the ticket price increase, quantity (number of tickets) sold goes down while revenue increases as predicted. But together with the rule change and larger jackpots, consumers’ utility in playing the game increases. This translates into increase real output (welfare) and accordingly the real price goes down, making the demand even more inelastic.

From the changes in output, price index and elasticity, we can conclude that the innovation of Lotto 6/49 not only simply changes the rules of the game, it also change the nature of the product.

## 5 Conclusion

In this paper, we study the demand for lottery tickets. We applied the non-expected utility theory to measure the outputs and prices of Lotto 6/49 in the one-dollar ticket and the two-dollar ticket games. The results show the nature of Lotto 6/49 changes with the ticket price and rule changes. The output of Lotto 6/49 have risen sharply while the price index fell dramatically. The output is six times higher than before while the price index is less than one sixth of the price index of the two-dollar period. Income elasticity decreased from 2.1 to 0.726. The output estimated by non-expected utility model is nine times higher than the official GDP in the two-dollar period while in the one-dollar period it is close to three times (Yu, 2004). The method used here can be used to other games such like Super 7. In fact, after the innovation the new Lotto 6/49 is more similar to Super 7, the output and price index of the Super 7 calculated by the  $N$ -states model may closer to those we get in two dollars period. In future studies it would be interesting to estimate and analyse the cross-price elasticity of Lotto 6/49 and Super 7.

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Table 1: Rules of Lotto 6/49

	Before June 2, 2004	After June 2, 2004
Ticket price	\$1	\$2
Take-out rate	55%	53%
Prize categories	5	6
Overall odds per ticket	1/54	1/32
<i>Share of the pool fund:</i>		
Jackpot	50%	80.5%
2nd prize	15%	5.75%
3rd prize	12%	4.75%
4th prize	23%	9%
5th prize	\$10	\$10
6th prize	N/A	\$5

Note: pool fund =  $(1 - \text{takeout rate}) \times \text{total wager} - \text{total 5th and 6th prizes pay-out}$

Table 2: Statistics of Lotto 6/49

	1 dollar	2 dollar	Change
Sales	15,541,318	18,585,285	19.6%
Pool fund	4,272,773	6,543,449	53.1%
Take-out	8,547,724	9,850,201	15.2%
Jackpot	2,136,386	5,267,476	146.6%
Jackpot winners	1.09	0.62	-43.1%
2nd prize	640,916	376,248	-41.3%
2nd winners	6.66	3.56	-46.6%
3rd prize	512,732	310,813	-39.4%
3rd winners	279	162	-41.9%
4th prize	982,737	588,910	-40.1%
4th winners	14,905	8,828	-40.8%
5th + 6th prize	2,720,820	2,191,635	-19.4%
5th + 6th winners	272,082	276,144	1.5%
5th winners	272,082	162,183	-40.4%

Table 3: Total Sales over Time				
	Coefficient	Standard Error	$p$ -value	$R^2$
<i>Lotto 6/49 total sales in 2 dollar game</i>				
Intercept	$3.63 \times 10^{10}$	$5.50 \times 10^6$	$2.75 \times 10^{-10}$	0.9989
Time	$1.82 \times 10^7$	39,107	0	
<i>Super 7 total sales</i>				
Intercept	$-8.02 \times 10^8$	$2.90 \times 10^7$	0	0.9849
Time	$1.96 \times 10^7$	$1.17 \times 10^5$	0	
<i>Lotto 6/49 total sales in 1 dollar game</i>				
Intercept	$2.67 \times 10^8$	$1.03 \times 10^7$	0	0.9979
Time	$1.55 \times 10^7$	27,960	0	

Table 4: Probability of winning for one ticket		
Prize	Rule	Probability of Winning, $\pi_i$
Jackpot	6 numbers	0.0000000715
Second	5 numbers + bonus	0.000000429
Third	5 numbers	0.00001802
Fourth	4 numbers	0.0009686
Fifth	3 numbers	0.01765
Sixth	2 numbers + bonus	0.0123

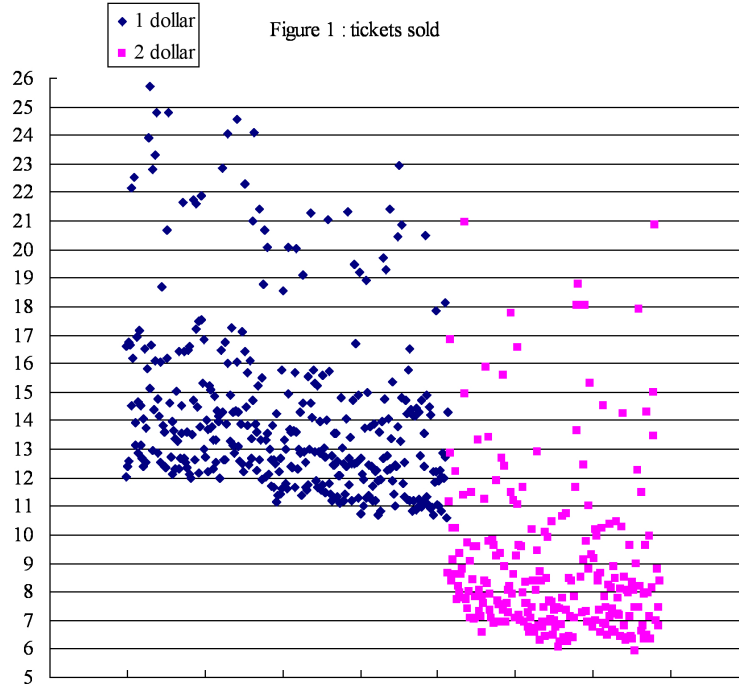


Figure 2: Sales Revenue

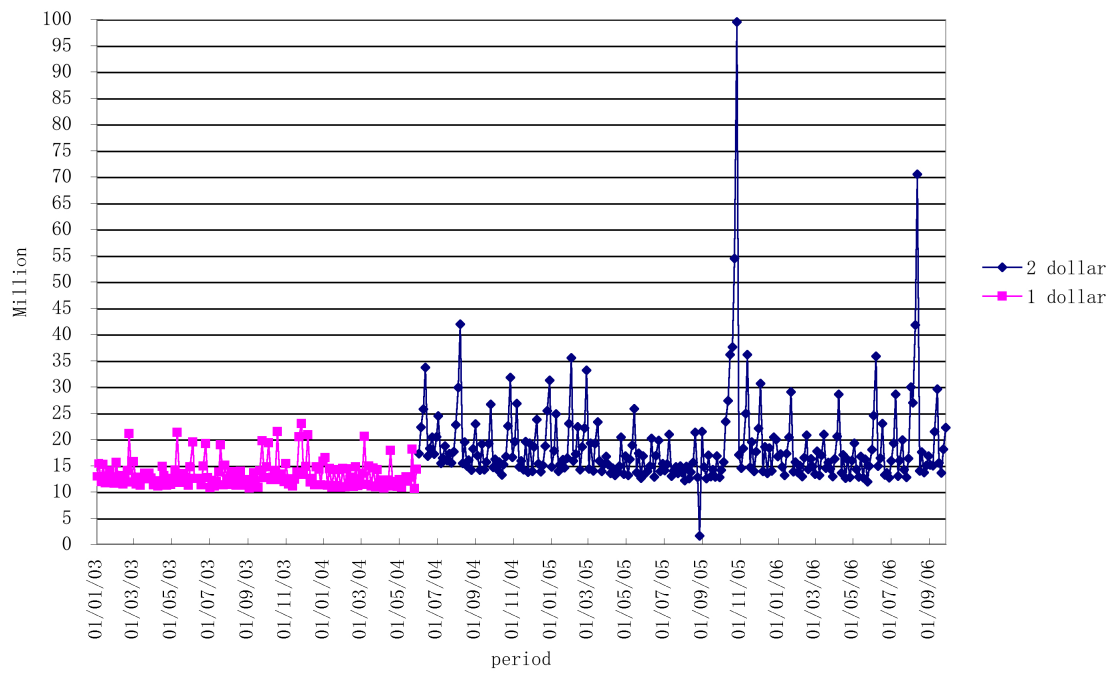


Figure 3: Super7 vs Lotto

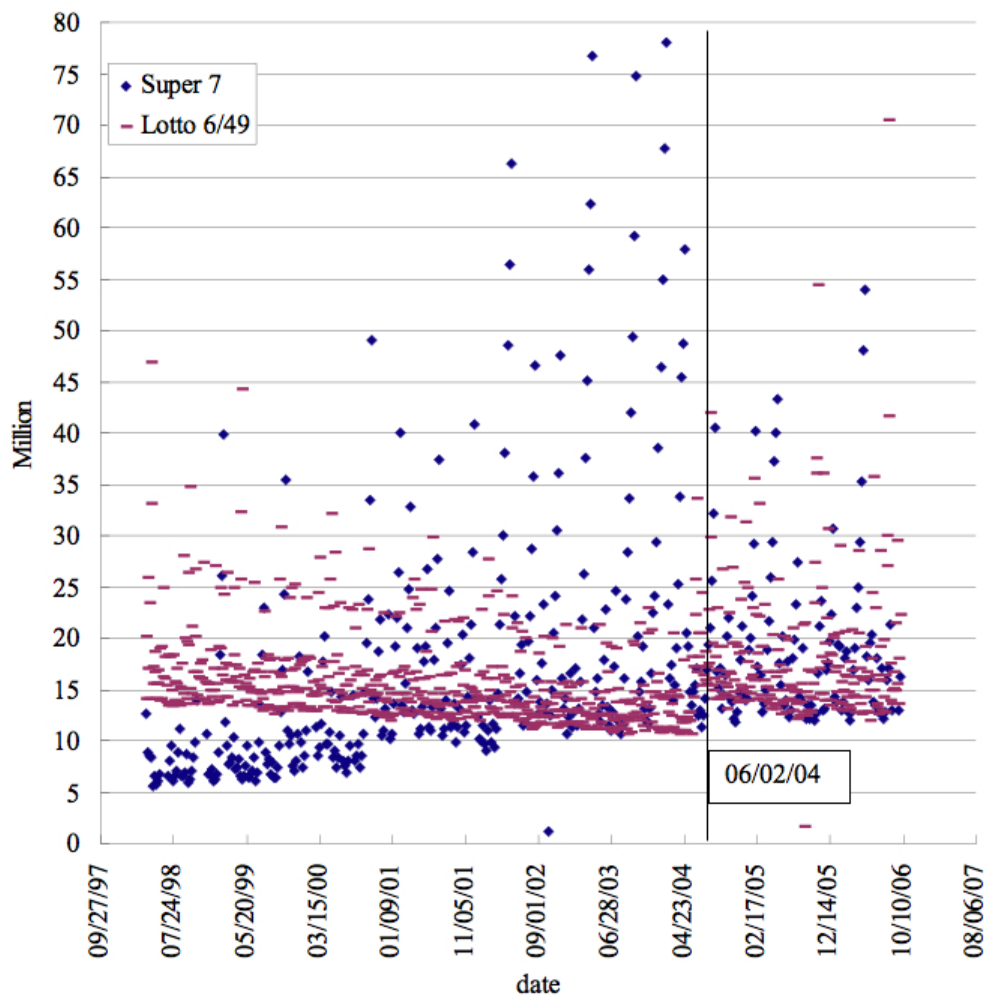




Figure 4: Output

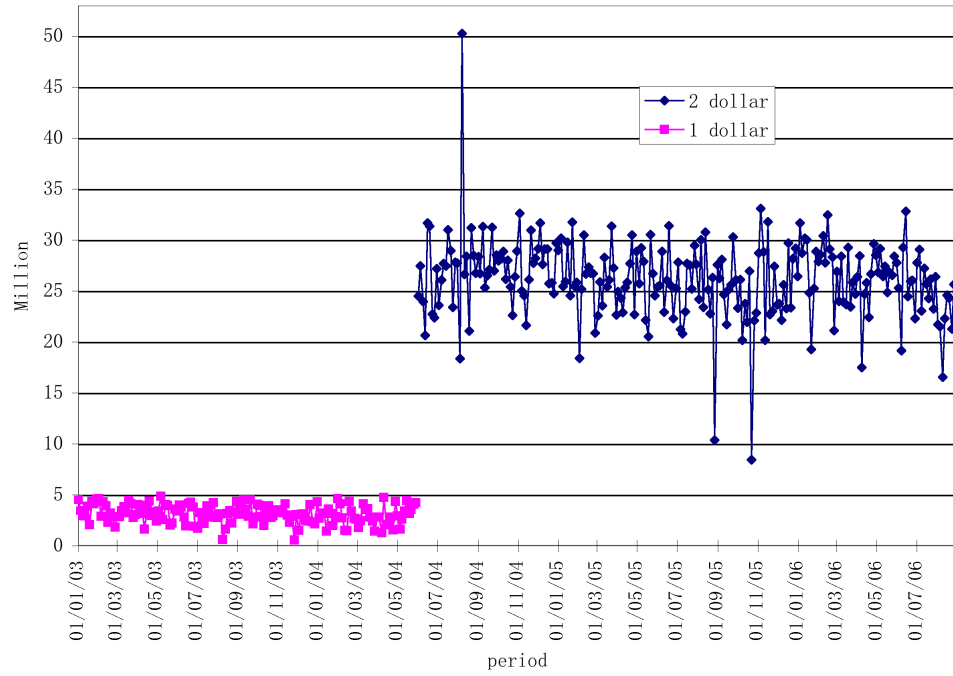


Figure 5: Productivity

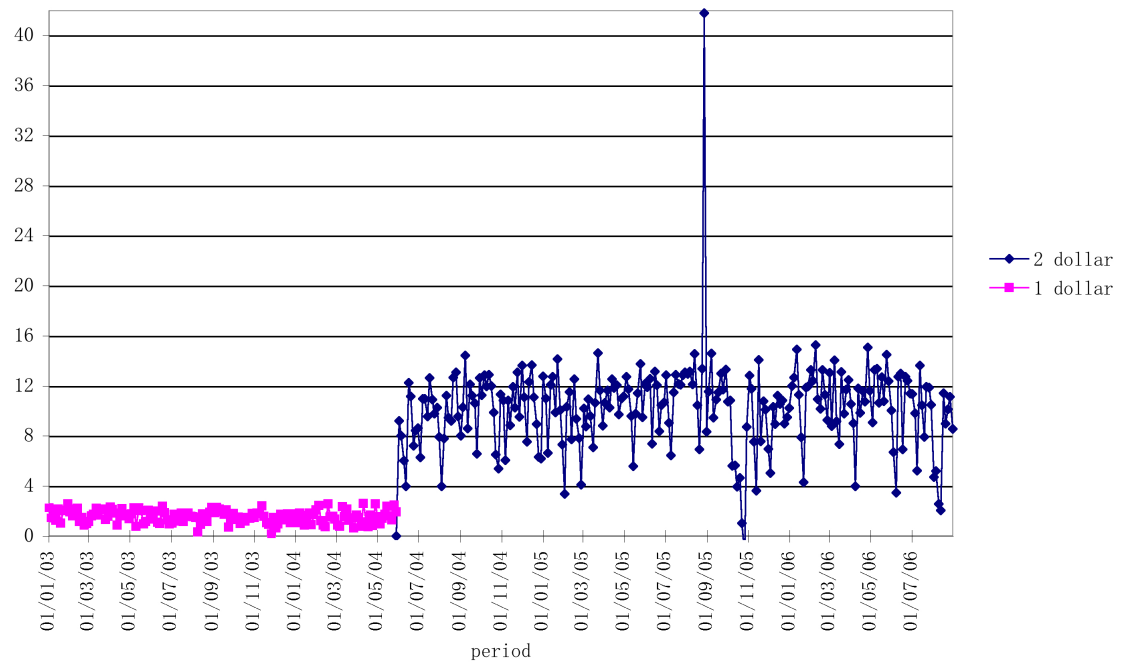


Figure 6: Price Index

